19

THE FRANKLIN INSTITUTE • *Laboratories for Research and Development*

Final Technical
Report F-A1914

A SPECIAL ANALYTICAL STUDY OF AIR-LUBRICATED BEARINGS FOR JET AIRCRAFT ENGINES

by
George M. Robinson

March 7, 1956 to February 15, 1957

Prepared for
National Advisory Committee for Aeronautics
Contract NAw6473

Copy No. 19

ABSTRACT

The analytical study of air lubricated bearings which is covered in this report had as its major objective the feasibility of adaptation of this type bearing to aircraft turbojet engine applications.

Design formulae and methods for a simple step bearing were derived. Bearing loads encountered in maneuvers listed in MIL-E-5007A were analyzed. These analyses and formulae indicate that air lubricated bearings may be adaptable to aircraft turbojet engines. However, large bearings, small clearances, and large volumes of air will be involved.

Further analyses, together with experimentation, are recommended to verify these conclusions.

TABLE OF CONTENTS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
1512 H STREET NORTHWEST
WASHINGTON 25, D. C.

TELEPHONE: LIBERTY 5-6700

LANGLEY AERONAUTICAL LABORATORY
LANGLEY FIELD, VA.

AMES AERONAUTICAL LABORATORY
MOFFETT FIELD, CALIF.

LEWIS FLIGHT PROPULSION LABORATORY
21000 BROOKPARK ROAD, CLEVELAND 11, OHIO

August 14, 1957

Mr. George P. Townsend, Jr.
Chief Engineer
Sundstrand Aviation - Denver
2480 West 70th Avenue
Denver 11, Colorado

Dear Mr. Townsend:

There is forwarded herewith, for your information and on loan, a copy of the following report which has been submitted by the Franklin Institute:

"A Special Analytical Study of Air-Lubricated Bearings for Jet Aircraft Engines," by George M. Robinson.

It would be appreciated if you would review this report and provide us with any comments you care to make for or against publication as a NACA Technical Note. In reply please refer to contract NAw-6473.

Very truly yours,

Harold F. Hipsher

Harold F. Hipsher
Secretary, NACA Subcommittee
on Lubrication and Wear

Enclosure
Copy No. 19 of the report

I. INTRODUCTION

On March 15, 1956, the staff of the Friction and Lubrication Section of The Franklin Institute began an analytical study to determine the feasibility of using gas-lubricated bearings on jet aircraft engines. Because of the fact that engine bearings are contemplated for use at higher loads, speeds, and ambient temperature and because the present bearing system has well defined limitations, it was thought that such a study should be undertaken.

Gas lubricated bearings have several inherent strong points and several handicaps. These bearings are clean, carry higher loads at higher temperatures, and do away with the necessity for seals. They are, however, relatively sensitive to overloads, have tendencies toward instability, and suffer all the problems attendant to high pressure flow.

The literature survey which was made at the beginning of this program confirmed our understanding that there was in existence no general theory for heavily loaded gas bearings. Before we could determine if the use of gas bearings was feasible, we had to first derive a theory which could be used in their analysis.

II. BEARING SPECIFICATIONS

When The Franklin Institute began work on this project, one of the first tasks was to determine a suitable set of bearing specifications for a jet engine. Consequently, visits to jet aircraft engine departments of United Aircraft Corporation, General Electric Company, and the Allison Division of General Motors Corporation were made and typical specifications were received from each. The following is a summary of the information obtained.

General Electric Company concentrates on single spool engines with three bearings. Allison Division of General Motors Corporation makes only a double spool engine with six bearings. Pratt & Whitney Aircraft

Company makes both a single spool engine with three bearings and a double spool engine with seven bearings but recommended that we concentrate on the single spool engine.

The bearing specifications submitted by Pratt & Whitney Aircraft Company were not explicit but were each reported as a constant multiplied by the shaft diameter squared. In order to compare Pratt & Whitney specifications with those of General Electric Company, four inches was chosen as a representative value for shaft diameter.

When this was done, the following specifications resulted:

Single Spool Engines:

No. 1 Bearing -

G.E. - 100 lb steady ± 400 lb oscillatory
7150 lb maneuver load for about 1 minute
P&W - 800 lb steady ± 7000 lb maneuver load

No. 2 Bearing - (radial load)

G.E. - 600 lb steady ± 2400 lb oscillatory
6000 lb maneuver load for about 1 minute
P&W - 960 lb steady ± 2320 lb maneuver load

No. 2 Bearing - (thrust load)

G.E. - 10,000 lb steady in the forward direction
4,000 lb steady in the aft direction
P&W - ± 6400 lb steady

No. 3 Bearing -

G.E. - 500 lb ± 2000 lb oscillatory
6000 lb maneuver load for one minute
P&W - 1150 lb ± 5360 lb maneuver load

Double Spool Engine (Allison):

Bearings No. 1 and No. 3 take thrust of 10,000 lb in either direction. Bearings No. 2, 4, 5, 6 have a steady radial load which ranges from 100 lb to 800 lb (mostly between 300 and 400 lb), transient loads of 10g, a 4g load for several minutes, and a gyro load of 4.0 rad/sec for 30 seconds varies up to 5 times the steady load. For sustained periods, a 4g + 1/2 rad/sec gyro load is about the worst condition that is encountered.

As a supplement to these specifications, we secured a copy of the military specifications (MIL-E-5007A) which cover turbojet aircraft engines. A preliminary reading of these military specifications indicated that, in order for the engine to withstand maneuvers specified in MIL-E-5007A, bearing loads would be much higher than given to us by the engine manufacturers. Consequently, additional data were obtained so that we could compute the bearing loads caused by the specified maneuvers.

In some cases, the computed loads proved to be as much as four times those given us by the engine manufacturers. For example, on an engine of the size considered on the preceding pages, maneuver loads as high as 19,000 lb were computed for the No. 3 bearing. When questioned about this point, the engine manufacturers told us that the bearing loads which they had originally given us were for the steady, plus oscillatory loads, plus those maneuver loads which were imposed for a relatively long period of time. The present rolling element bearings are selected on the basis of these loads. If the bearing which is selected has a static load carrying capacity greater than the more transient bearing loads, the bearing is assumed to be satisfactory.

While it is known that rolling element bearings can sustain overloads without failure for short periods of time, the effects of the magnitude, frequency, and duration of these overloads on the life of these bearings is not known. A program to determine these effects might prove to be quite useful.

Plane bearings can, in general, absorb a transient overload by virtue of their squeeze-film action. The ability of a lubricant to absorb suddenly applied loads is, however, directly proportional to its viscosity. Since the viscosity of air is extremely low, its ability to absorb transient loads by squeeze-film lubrication is almost nil.

Because of this fact, we next set about to determine how realistic the specifications in MIL-E-5007A were. For example, section 3.15 states: "At maximum rated engine speed, the engine shall withstand a gyroscopic moment imposed by a steady angular velocity of 4.0 radians per second in yaw, combined with a vertical load factor of $\pm$ 1, for a period of 30 seconds." It seemed to us that this requirement was somewhat excessive. It is difficult to conceive of some of our modern aircraft yawing at a rate of 64% of a complete revolution a second and keeping it up for 30 seconds. I checked with several agencies at the Airforce, Navy Department, etc., and could not find anyone who could tell me how realistic this specification was. Several pilots, with whom I spoke, told me that such a sustained value of angular velocity is much too high. They agreed that the maximum average yaw velocity ever to be encountered would be about one revolution in five seconds (1.25 rad/sec). They also stated that a plane spins like a falling leaf and that, during such a maneuver, instantaneous yaw velocities may reach 4.0 rad/sec and indeed exceed it. At any rate, the maximum value of yaw velocity is not known.

Since the bearing loads are directly proportional to the moment of the gyroscopic couple during a yaw maneuver, they are also proportional to the angular velocity of precession in yaw of the aircraft. Since the maximum value of this precessional velocity is not known, the maximum bearing loads are also unknown. As was pointed out in a previous section, air bearings as a rule are quite sensitive to overloading. It is, therefore, impossible to design air bearings for this application with a known degree of safety. If air bearings are to be used, they must be overdesigned somewhat so that some degree of safety factor may be realized.

Another problem which presents itself is the fact that the bearing must take heavy loads in all directions. It becomes obvious, then, that we shall need several hydrostatic air bearings around the periphery of the journal. Because each bearing is loaded by other of the bearings in this system, it must be capable of absorbing this additional loading as well as its share of the normal bearing loads. If all of these individual bearings are approximately equal in load carrying capacity under like conditions, the entire bearing (which is composed of all the individual bearings) must then depend on eccentricity of the shaft within the bearing for its load carrying capacity. This, in turn, means that the bearing is completely dependent upon a competently designed throttling system. The design of such a throttling system becomes a major problem in the analysis.

III. ANALYTICAL APPROACH

The first phase of this part of the program was devoted to a literature survey to determine the state of the art for the design of air bearings. The results of this survey confirmed our experience that the state of the art is still in its childhood, if indeed not its infancy.

It was found that while there were several rather simplified analytical methods for designing hydrostatic air bearings, there was insufficient experimental proof to back up any one method. Some of the theories ignored momentum effects entirely, some considered viscous friction, some considered skin friction only. In only a few was there mention of the possibility of sonic or supersonic flow in a bearing, or choking in the capillaries or orifices leading to the bearings.

It was necessary, then, to combine what we thought were the best characteristics of each method, determine which of the simplified methods applied and, then, use that simplified method to determine the feasibility of the bearings for jet engine application.

IV. DERIVATION OF NEEDED EQUATIONS

A. Viscous Flow Through a Capillary

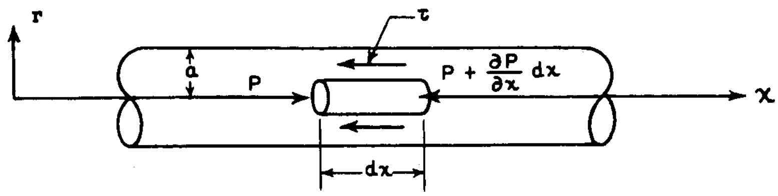


FIGURE 1

Assumptions:

The flow is subsonic and laminar
momentum effects are neglected.
Air behaves as a perfect gas.

Let

G = mass flow rate (lb-sec/in.)

P = pressure (psi)

τ = shear stress (psi)

r = radius of element of fluid (in.)

a = outer radius of capillary

v = velocity of air at radius r

μ = absolute viscosity of fluid (lb-sec/in.^{2})

ℓ = length of capillary (in.)

R = gas constant

T = temperature of air in degrees Rankine

ρ = mass density of air (slugs/ft^3)

h = film thickness of step bearing (in.)

R_o = inner radius of step bearing (in.)

R_a = outer radius of step bearing (in.)

y = P/P_o where P_o is pressure at R_o

x = r/R_o

$W = x^2 y^{(1/m+1)}$

m = exponent in expansion process p/ρ^m = constant

k = exponent in isentropic expansion process p/ρ^k = constant

C = local sonic velocity = $(\partial p/\partial \rho)_s = \sqrt{k\ p/\rho}$

V_c = velocity of air entering orifice

V_o = velocity of air leaving orifice

M = local Mach number = V/C

Let us consider the forces on the elemental volume $\pi r^2 dx$ in the x direction:

The only force to the right is $P(\pi r^2)$

The forces to the left are $(P + \partial P/\partial x\ dx)\ \pi r^2 + \tau(2\pi r dx)$

If we sum up all the forces in the x direction and set the resulting expression equal to zero we find:

$$-\frac{\partial P}{\partial x} dx \cdot \pi r^2 - 2\tau\pi r dx = 0$$

$$r \frac{\partial P}{\partial x} = -2\tau$$

We will now make a further assumption that P varies only with x so that $\partial P/\partial x = dP/dx$. Therefore,

$$r \frac{dP}{dx} = -2\tau = -2\mu \frac{dv}{dr}$$

$$dv = \frac{1}{2\mu} \frac{dP}{dx} r dr = dv$$

$$v = \frac{1}{2\mu} \frac{dP}{dx} \frac{r^2}{2} + C$$

When $r = a$, $v = 0$

$$C = -\frac{1}{2\mu} \frac{dP}{dx} \frac{a^2}{2}$$

$$v = \frac{1}{2\mu}\frac{dP}{dx}\frac{(r^2 - a^2)}{2}$$

$v_{maximum}$ occurs when r equals zero

$$v_{max} = -\frac{a^2}{4\mu}\frac{dP}{dx}$$

v_{avg} for a paraboloid is 1/2 Vmax. Therefore,

$$V_{avg} = -\frac{a^2}{8\mu}\frac{dP}{dx}$$

The mass flow rate $G = \rho AV_{avg}$. Therefore,

$$G = -\rho\pi a^2 \cdot \frac{a^2}{8\mu}\frac{dP}{dx} = -\frac{\rho\pi a^4}{8\mu dx}\,dP$$

$$dx = -\frac{\rho\pi a^4}{8\mu G}\,dP$$

but

$$\rho = \frac{P}{RT}$$

therefore

$$dx = -\frac{p\pi a^4}{8\mu GRT}\,dP$$

For the condition where T is a constant

$$x = -\frac{P^2\pi a^4}{16\mu GRT} + C$$

but when $x = 0$, $P = P_o$. Therefore,

$$C = \frac{\pi a^4 P_o^2}{16\mu RTG}$$

Therefore

$$x = \frac{\pi a^4}{16\mu RTG}\left[P_o^2 - P^2\right] \tag{1}$$

When $x = \ell$, $P = P_a$. Therefore,

$$G = \frac{\pi a^4}{16\mu RT\ell}\left[P_o^2 - P_a^2\right] \tag{2}$$

Let us now consider the effect of momentum changes on Equation (1).

From the previous analysis we found that the sum of the external forces in the x direction was

$$-\frac{\partial P}{\partial x}\, dx\pi r^2 - 2\tau\pi r dx$$

The summation of the external forces on the elementary control volume is equal to the time rate of change of momentum in the x direction of the fluid within the control volume. We should remember that both $\partial P/\partial x$ and dV/dr are both negative. This may be important later in determining proper signs in the final equation.

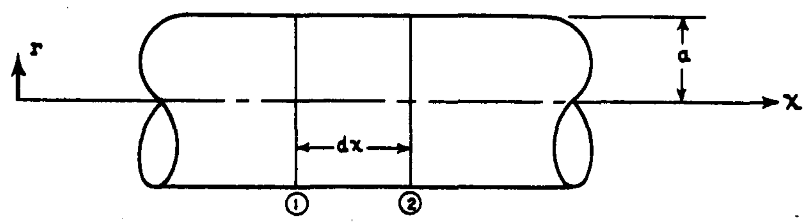


FIGURE 2

At section one in Figure 2, the total x momentum entering is

$$\int_0^a 2\pi r \cdot \rho V^2 dr$$

If we assume that $\partial\ell/\partial r = 0$, then this expression becomes

$$2\pi \underset{\rho}{P} \int_0^a V^2 r dr = 2\pi\rho V_{rms}^2 \int_0^a r dr = \pi\rho V_{rms}^2\, a^2$$

at Section 2, the total x momentum leaving is

$$\rho V_{rms}^2 \pi a^2 + \frac{\partial}{\partial x}(\rho V_{rms}^2 \pi a^2)dx$$

The time rate of change of x momentum for the control volume of Figure 2 is

$$\frac{\partial}{\partial x}(\rho V_{rms}^2 \pi a^2)$$

The time rate of change of x momentum for the elementary control volume of Figure 1 is

$$\frac{r^2}{a^2}\frac{\partial}{\partial x}(\rho V_{rms}^2 \pi a^2)dx$$

but
$$\rho V_{rms}^2 \pi a^2 = \frac{\rho^2 V_{rms}^2 \pi^2 a^4}{\rho \pi a^2} = \frac{G^2}{k\rho\pi a^2}$$

where
$$k = \frac{V_{rms}^2}{V_{avg.}^2}$$

but $\rho = P/RT$. Therefore,

$$\frac{G^2}{k\rho\pi a^2} = \frac{G^2 RT}{kP\pi a^2}$$

therefore,
$$\frac{r^2}{a^2}\frac{\partial}{\partial x}(\rho V_{rms}\pi a^2)dx = \frac{r^2}{a^2}\frac{\partial}{\partial x}\left(\frac{G^2 RT}{kP\pi a^2}\right)dx$$

therefore,
$$-\frac{\partial P}{\partial x}dx \cdot \pi r^2 - 2\tau\pi r dx = \frac{r^2}{a^2}\frac{\partial}{\partial x}\left(\frac{G^2 RT}{k^2 P\pi a^2}\right)dx$$

$$-\frac{\partial P}{\partial x} - \frac{2\tau}{r} = \frac{1}{\pi a^2}\frac{\partial}{\partial x}\left(\frac{G^2 RT}{kP\pi a^2}\right)$$

$$\frac{2\tau}{r} = -\frac{\partial P}{\partial x} - \frac{1}{\pi a^2}\frac{\partial}{\partial x}\left(\frac{G^2RT}{P\pi a^2 g}\right)$$

but
$$\tau = -\mu\frac{dv}{dr}, \quad \frac{\partial P}{\partial x} = \frac{dP}{dx}$$

Therefore,
$$-\frac{2\mu}{r}\frac{dv}{dr} = -\left[\frac{dP}{dx} + \frac{1}{\pi a^2}\frac{d}{dx}\left(\frac{G^2RT}{kP\pi a^2}\right)\right]$$

$$dv = \frac{1}{2\mu}\left[\frac{dP}{dx} + \frac{1}{\pi a^2}\frac{d}{dx}\left(\frac{G^2RT}{kP\pi a^2}\right)\right] rdr$$

$$v = \frac{1}{2\mu}\left[\frac{dP}{dx} + \frac{1}{\pi a^2}\frac{d}{dx}\left(\frac{G^2RT}{kP\pi a^2}\right)\right]\frac{r^2}{2} + C$$

when $r = a$, $v = 0$. Therefore,

$$v = \frac{1}{2\mu}\left[\frac{dP}{dx} + \frac{1}{\pi a^2}\left(\frac{d}{dx}\frac{G^2RT}{kP\pi a^2}\right)\right]\left(\frac{r^2 - a^2}{2}\right)$$

Avg $v = \bar{v} = 1/2\ V_{max} = V_{rms}/2\sqrt{1.2}$ Therefore,

$$\bar{v} = -\frac{a^2}{8\mu}\left[\frac{dP}{dx} + \frac{1}{\pi a^2}\frac{d}{dx}\left(\frac{1.2\ G^2RT}{P\pi a^2}\right)\right]$$

but
$$G = \rho A\bar{v} = -\rho\pi a^2 \cdot \frac{a^2}{8\mu}\left[\frac{dP}{dx} + \frac{1}{\pi a^2}\frac{d}{dx}\left(\frac{1.2\ G^2RT}{P\pi a^2}\right)\right]$$

$$G = -\frac{\rho\pi a^4}{8\mu}\left[\frac{dP}{dx} + \frac{1}{\pi a^2}\frac{d}{dx}\left(\frac{1.2\ G^2RT}{P\pi a^2}\right)\right]$$

but $\rho = P/RT$. Therefore,

$$G = \frac{P\pi a^4}{8\mu RT}\left[\frac{dP}{dx} + \frac{1}{\pi a^2}\frac{d}{dx}\left(\frac{1.2\ G^2RT}{P\pi a^2}\right)\right]$$

$$G = -\frac{\pi a^4}{8\mu RT}\left[\frac{Pdp}{dx} + \frac{1.2\ PG^2RT}{\pi^2 a^4}\frac{d}{dx}\left(\frac{1}{P}\right)\right]$$

$$G = -\frac{\pi a^4}{8\mu RT}\left[\frac{Pdp}{dx} - \frac{1.2\ G^2RTP}{\pi^2 a^4 P^2}\frac{dP}{dx}\right]$$

$$G = -\frac{\pi a^4 Pdp}{8\mu RTdx} + \frac{1.2\ G^2dP}{8\mu\pi Pdx}$$

$$dx = -\frac{\pi a^4 Pdp}{8\mu GRT} + \frac{1.2\ G}{8\mu\pi}\frac{dP}{P}$$

for the condition wherein T is a constant.

ln P

$$x = -\frac{\pi a^4 P^2}{16\mu GRT} + \frac{1.2\ G\ell nP}{8\mu\pi} + C$$

When $x = 0$, $P = P_o$. Therefore,

ln

$$x = \frac{\pi a^4}{16\mu GRT}\left[P_o^2 - P^2\right] - \frac{1.2\ G}{8\pi\mu}\ell n\frac{P_o}{P} \qquad (3)$$

$x = \ell$, $P = P_a$.

$$\ell = \frac{\pi a^4}{16RT\mu G}\left(P_o^2 - P_a^2\right) - \frac{1.2\ G}{8\pi\mu}\ell n\frac{P_o}{P_a} \qquad (4)$$

Let us consider a typical problem.

Let $P_o = 100$ psia

$R = 53.3 \times 12\ \frac{in.lb}{lb°R} \times 386\ \frac{in.}{sec^2} = 2.48 \times 10^5\ \frac{in.^2}{sec^2 °R}$

$T = 530°R$

$\mu = 2.5 \times 10^{-9}\ \frac{lb\text{-}sec}{in.}$

$G = 5.18 \times 10^{-6}\ \frac{lb\text{-}sec}{in.}$

$$a = 30 \times 10^{-3} \text{ in.}$$
$$a^4 = 8.1 \times 10^{-7} \text{ in.}^4$$

$$\frac{\pi a^4}{16RT\mu G} = \frac{3.14 \times 8.1 \times 10^{-7}}{16 \times 2.48 \times 10^5 \times 530 \times 2.5 \times 10^{-9} \times 5.18 \times 10^{-6}} = 0.0934$$

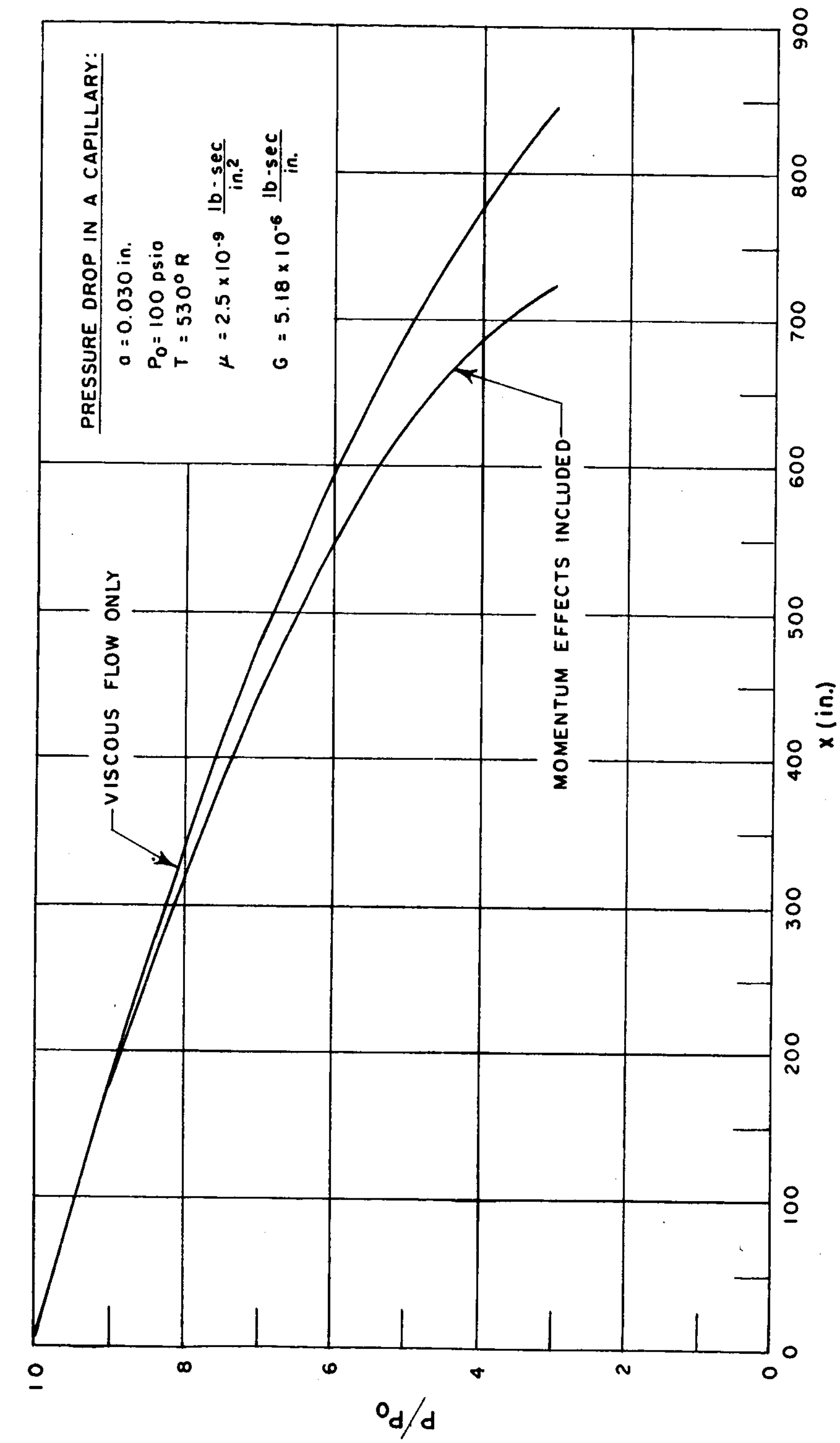
$$\frac{1.2\ G}{8\mu\pi} = \frac{1.2 \times 5.18 \times 10^{-6}}{8 \times 2.5 \times 10^{-9} \times 3.14} = 99.1$$

Therefore, $x = 0.0934\ (P_o^2 - P^2) - 99.1\ \ln P_o/P$

$$x = 0.0934\ P_o^2\ (1 - P^2/P_o^2) - 99.1\ \ln P_o/P$$

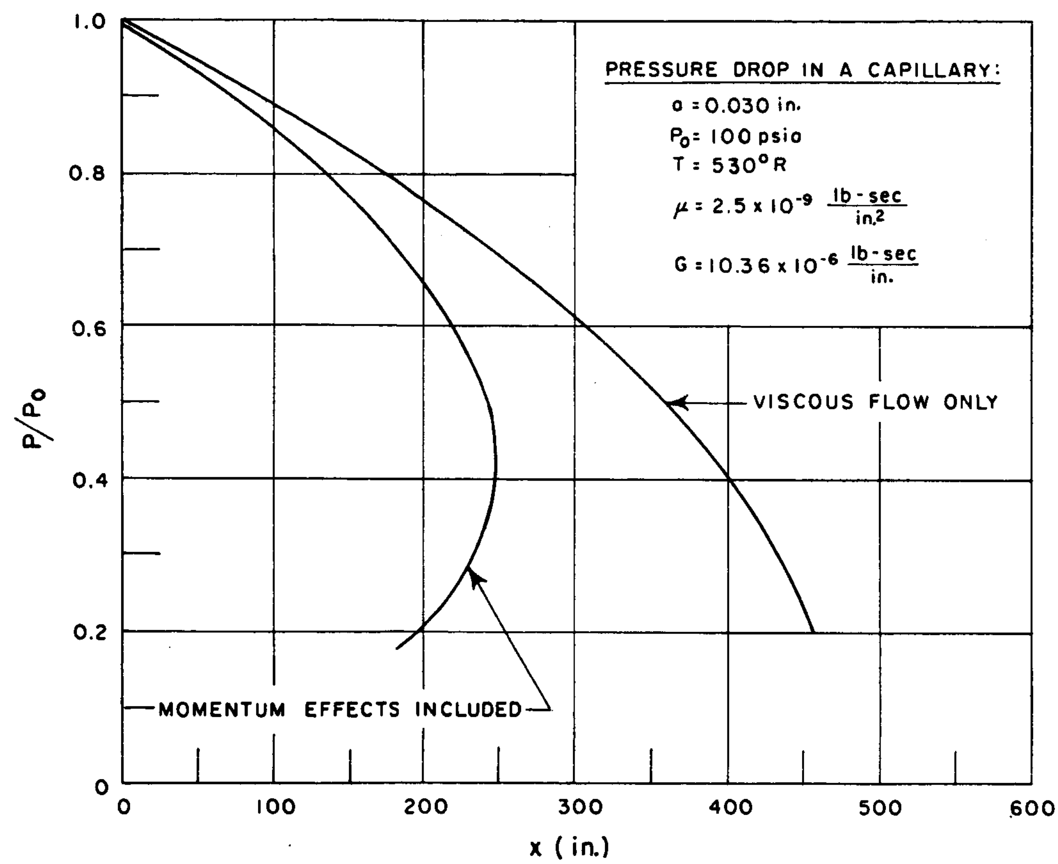
Figure 3 is a plot of P/P_o vs. x for this equation. For comparison, the same values of parameters were used in Equation (1), and the results plotted. We can now readily see the relative importance of the momentum and viscous effects. In this case, the momentum effects are not important for a considerable distance along the capillary. A subsequent check of the Mach number at the entrance disclosed that this Mach number was low ($M \approx 0.15$).

In order to illustrate the relative effects of viscous and momentum effects, Figure 4 was plotted. In this case, all specifications of the preceding problem were unchanged but the flow. It was doubled. We see now that momentum effects are important throughout the entire length of the capillary. It becomes apparent that before one decides to use the simplified equation, one should check the initial and final Mach numbers to determine if any appreciable error will be involved.



REPORT F-A1914

FIGURE 3


PRESSURE DROP IN A CAPILLARY:
a = 0.030 in.
P_0 = 100 psia
T = 530°R
$\mu = 2.5 \times 10^{-9}\ \frac{\text{lb-sec}}{\text{in.}^2}$
$G = 10.36 \times 10^{-6}\ \frac{\text{lb-sec}}{\text{in.}}$
VISCOUS FLOW ONLY
MOMENTUM EFFECTS INCLUDED
P/P_0
1.0
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
x (in.)

B. Viscous Flow Through A Simple Step Bearing

We are going to restrict ourselves to an analysis of a simple step bearing for several reasons. First, it will adequately substantiate or disprove the feasibility of gas bearings for the particular application. Secondly, it lends itself to a relatively simple analysis. A typical step bearing is illustrated in Figure 5.

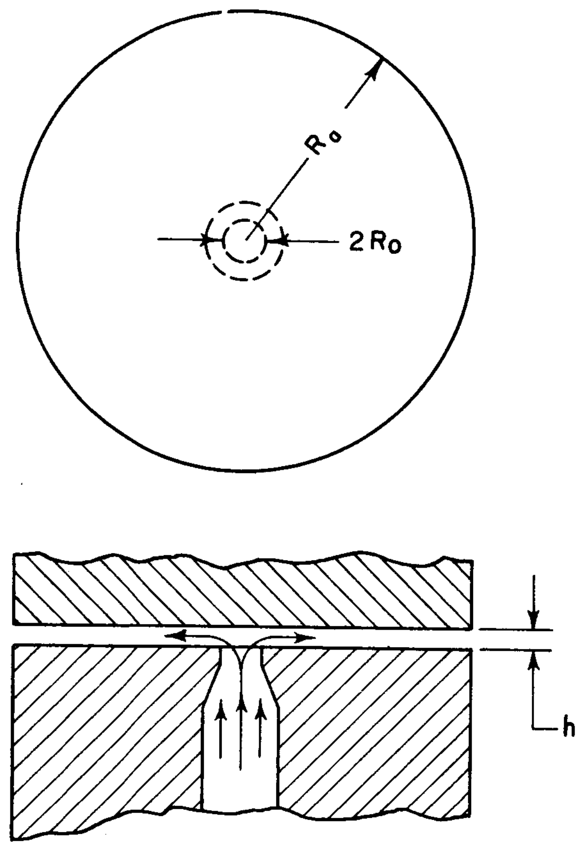


FIGURE 5

An an externally pressurized step bearing, air under pressure is introduced though a capillary and/or orifice between the bearing surfaces. It is this externally pressurized supply of air which actually forces the bearing surfaces apart and makes the bearing work.

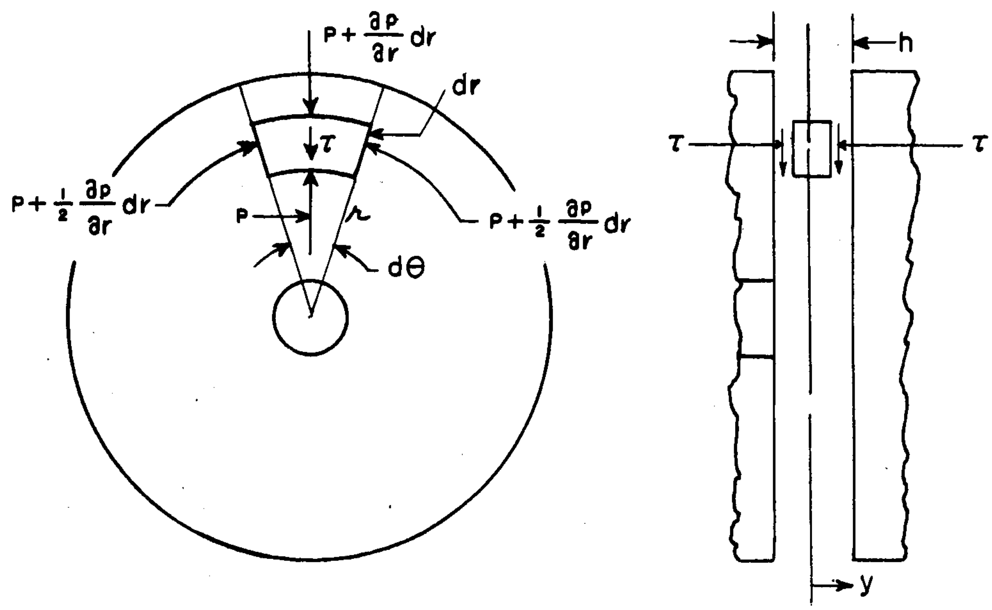


FIGURE 6

1. Viscous Flow Through a Radial Step Bearing, Momentum Effects Neglected.

Consider the Forces on the elemental volume of air shown in Figure 6.

$$\Sigma Fr = 0 = 2y\frac{\partial P}{\partial r}\,dr(rd\theta) + 2\tau rd\theta dr = 0, \quad \tau = \mu\frac{dv}{dy}$$

$$y\frac{dP}{dr} - \mu\frac{dv}{dy} = 0$$

$$\frac{dP}{dr} = \frac{\mu}{y}\,\frac{dv}{dy}$$

$$dv = \frac{ydy}{\mu}\frac{dP}{dr}$$

$$v = \frac{1}{\mu}\frac{dP}{dr}\frac{y^2}{2} + C$$

when

$$y = \frac{h}{2}, \ v = 0$$

$$v = \frac{1}{\mu}\frac{dP}{dr}\left[\frac{y^2}{2} - \frac{h^2}{8}\right]$$

$$v_{max} = -\frac{1}{\mu}\frac{dP}{dr}\left(\frac{h^2}{8}\right)$$

$$v_{avg.} = \frac{2}{3}V_{max} = -\frac{1}{12\mu}\frac{\partial P}{\partial r}h^2$$

$$G = \rho A v_{avg} = -\rho.2\pi rh \cdot \frac{1}{12\mu}\frac{\partial P}{\partial r}h^2 = -\frac{\pi rh^3}{6\mu}\frac{\partial P}{\partial r}\cdot\rho \text{ but } \rho = \frac{P}{RT}$$

$$G = -\frac{\pi rh^3}{6\mu RT}\frac{PdP}{dr}$$

$$\frac{dr}{r} = -\frac{\pi h^3}{6\mu RTG}PdP$$

$$\ell n r = -\frac{\pi h^3}{6\mu RTG}\frac{P^2}{2} + C \qquad \text{when } r = R_o, \ P = P_o$$

$$C = \ell n R_o + \frac{\pi h^3}{6\mu RTG}\frac{P_o^{\ 2}}{2}$$

Therefore

$$\ell n \frac{r}{R_o} = \frac{\pi h^3(P_o^2 - P^2)}{12\mu RTG}$$

$$G = \frac{\pi h^3 (P_o^2 - P^2)}{12\mu RT \ln r/R_o} \tag{5}$$

2. Viscous Flow Through a Radial Step Bearing, Momentum Effects Included

The sum of the external forces on the control volume in question in the radial direction is equal to the net time rate of efflux of momentum from the control volume in the r direction.

The sum of the external forces on the control volume in question in the r direction is found from the previous analysis to be

$$-2y\frac{\partial P}{\partial r}\, dr\, (r d\theta) - 2\tau r d\theta dr$$

The time rate of change of r momentum within the elementary control volume is

$$\frac{2y}{h}\frac{\partial}{\partial r}\left[\rho r d\,\theta V_{rms}^2\, h\right] dr$$

Therefore

$$-2y\frac{\partial P}{\partial r}\, dr\, (r d\theta) - 2\tau r d\theta dr = 2y d\theta \frac{\partial}{\partial r}\left[\rho V_{rms}^2 r\right] dr$$

$$-\frac{\partial P}{\partial r} - \frac{\tau}{y} = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(\rho V_{rms}^2 r\right)\right]$$

If we assume a parabolic distribution of velocity in the y direction, $V_{rms}^2 = 1.2\, V_{avg}^2$

If $G^2 = \rho^2 A^2 V_{avg}^2 = \dfrac{\rho^2 A^2 V_{rms}^2}{1.2}$

then $V_{rms}^2 = \dfrac{1.2 G^2}{\rho^2 A^2}$

Therefore,

$$-\frac{\partial P}{\partial r} - \frac{\tau}{y} = \frac{1}{r}\left[\frac{\partial}{\partial r}\,\rho . r\,\frac{1.2G^2}{\rho^2(2\pi rh)^2}\right]$$

$$\frac{\partial P}{\partial r} = \frac{dP}{dr}$$

$$-\frac{dP}{dr} - \frac{\tau}{y} = \frac{1.2G^2}{4\pi^2 rh^2}\,\frac{d}{dr}\left[\frac{1}{\rho r}\right]$$

$$\frac{\tau}{y} = -\frac{\mu}{y}\frac{dV}{dy} = -\frac{dP}{dr} - \frac{1.2G^2}{4\pi^2 rh^2}\,\frac{d}{dr}\left[\frac{1}{\rho r}\right]$$

$$dV = \frac{1}{\mu}\left[\frac{dP}{dr} + \frac{1.2G^2}{4r\pi^2 h^2}\,\frac{d}{dr}\left(\frac{1}{\rho r}\right)\right]\ ydy$$

$$V = \frac{1}{\mu}\left[\frac{dP}{dr} + \frac{1.2G^2}{4r\pi^2 h^2}\,\frac{d}{dr}\left(\frac{1}{\rho r}\right)\right]\frac{y^2}{2} + C$$

when

$$y = \frac{h}{2},\ V = 0$$

Therefore,

$$C = -\frac{1}{\mu}\left\{\frac{dP}{dr} + \frac{1.2G^2}{4r\pi^2 h^2}\,\frac{\partial}{dr}\left(\frac{1}{\rho r}\right)\right\}\frac{h^2}{8}$$

Therefore,

$$V = \frac{1}{\mu}\left\{\frac{dP}{dr} + \frac{1.2G^2}{4\pi^2 rh^2}\,\frac{d}{dr}\left(\frac{1}{\rho r}\right)\right\}\left\{\frac{y^2}{2} - \frac{h^2}{8}\right\}$$

$$V_{max} = -\frac{1}{\mu}\left\{\frac{dP}{dr} + \frac{1.2G^2}{4\pi^2 rh^2}\,\frac{d}{dr}\left(\frac{1}{\rho r}\right)\right\}\left\{\frac{h^2}{8}\right\}$$

$$V_{avg} = - \frac{h^2}{12\mu}\left\{\frac{dP}{dr} + \frac{1.2G^2}{4\pi^2 rh^2}\ \frac{d}{dr}\left(\frac{1}{\rho r}\right)\right\}$$

$$G = \rho AV_{avg.} = - \rho\, 2\pi rh \cdot \frac{h^2}{12\mu}\left\{\frac{dP}{dr} + \frac{1.2G^2}{4\pi^2 rh^2}\ \frac{d}{dr}\left(\frac{1}{\rho r}\right)\right\}$$

$$1 = - \frac{\pi r\rho h^3}{6\mu G}\left\{\frac{dP}{dr} + \frac{1.2G^2}{4\pi^2 rh^2}\ \frac{d}{dr}\left(\frac{1}{\rho r}\right)\right\}$$

$$1 = - \frac{\pi r\rho h^3}{6\mu G}\left\{\frac{dP}{dr} - \frac{1.2G^2}{4\pi^2 h^2 \rho r^3} - \frac{1.2\,G^2}{4\pi^2 h^2 \rho^2 r^2}\ \frac{d\rho}{dr}\right\}$$

$$\frac{6\mu G}{\pi r\rho h^3} = \frac{1.2G^2}{4\pi^2 h^2 \rho r^3} + \frac{1.2G^2}{4\pi^2 h^2 \rho^2 r^2}\ \frac{d\rho}{dr} - \frac{dP}{dr}$$

$$\text{but } \frac{P}{P_o} = \left(\frac{\rho}{\rho_o}\right)^m, \quad \frac{\rho}{\rho_o} = \left(\frac{P}{P_o}\right)^{1/m}, \quad \rho = \rho_o\left(\frac{P}{P_o}\right)^{1/m}$$

$$\frac{d\rho}{dr} = \frac{\rho_o}{P_o}\left(\frac{P}{P_o}\right)^{(1/m)-1} \cdot \frac{1}{m}\ \frac{dP}{dr}$$

Therefore

$$\frac{6\mu G}{\rho\pi rh^3} = \frac{1.2G^2}{4\pi^2 h^2 \rho r^3} + \frac{1.2G^2}{4\pi^2 h^2 \rho^2 r^2}\ \frac{\rho_o}{P_o}\left(\frac{P}{P_o}\right)^{(1/m)-1} \cdot \frac{1}{m}\ \frac{dP}{dr} - \frac{dP}{dr}$$

$G = 2\pi R_o V_o \rho_o h$ where V_o is the average Velocity at R_o

$$\rho = \rho_o \frac{P}{P_o}^{1/m}$$

Therefore

$$\frac{6\mu, 2\pi R_o h \rho_o V_o}{\rho_o \frac{P}{P_o}^{1/m} \pi r h^3} = \frac{1.2 \times 4\pi^2 R_o^2 \rho_o^2 h^2 V_o^2}{4\pi^2 h^2 \rho_o \frac{P}{P_o}^{1/m} r^3} + \left[\frac{1.2 R_o^2 V_o^2 \rho_o^2}{\rho_o^2 \frac{P}{P_o}^{2/m} r^2 m} \frac{\rho_o}{P_o} \left(\frac{P}{P_o}\right)^{(1/m)-1} - 1 \right] \frac{dP}{dr}$$

$$\frac{12\mu R_o V_o}{\left(\frac{P}{P_o}\right)^{1/m} r h^2} = \frac{1.2 \rho_o R_o^2 V_o^2}{r^3 \left(\frac{P}{P_o}\right)^{1/m}} + \frac{1.2 R_o^2 V_o^2}{m \left(\frac{P}{P_o}\right)^{(1/m)+1} r^2} \frac{\rho_o}{P_o} \frac{dP}{dr} - \frac{dP}{dr}$$

$$\frac{12\mu R_o V_o}{h^2} - \frac{1.2 \rho_o R_o^2 V_o^2}{r^2} = \left[\frac{1.2 R_o^2 V_o^2 \rho_o}{r^2 P_o m \left(\frac{P}{P_o}\right)} - \left(\frac{P}{P_o}\right)^{1/m} \right] r \frac{dP}{dr}$$

$$\frac{dP}{dr} = \frac{1}{r} \frac{\left[\frac{12\mu R_o V_o}{h^2} - \frac{1.2 \rho_o R_o^2 V_o^2}{r^2} \right]}{\left[\frac{1.2 R_o^2 \rho_o V_o^2}{r^2 P_o m \left(\frac{P}{P_o}\right)} - \left(\frac{P}{P_o}\right)^{1/m} \right]}$$

$$\frac{dP}{dr} = \frac{1}{r} \frac{\left[12\mu \frac{R_o V_o}{h^2} \frac{r^2}{R_o^2} - 1.2\rho_o V_o^2\right]}{\left[\frac{1.2\rho_o V_o^2}{P_o m \left(\frac{P}{P_o}\right)} - \frac{r^2}{R_o^2}\left(\frac{P}{P_o}\right)\right]^{1/m}}$$

$$\frac{dP}{dr} = \frac{P}{r} \frac{\left[\frac{12\mu P_o V_o}{h^2 P_o} \frac{r^2}{R_o^2} - \frac{1.2\rho_o V_o^2}{P_o}\right]}{\left[\frac{1.2\rho_o V_o^2}{m P_o} - \frac{r^2}{R_o^2}\left(\frac{P}{P_o}\right)\right]^{(1/m)+1}}$$

Let $x = \frac{r}{R_o}$, $y = \frac{P}{P_o}$, $w^2 = 12\mu \frac{R_o V_o}{h^2 P_o}$

$$P = yP_o \qquad r = R_o dx$$

$$dP = P_o dy$$

$$\frac{dP}{dr} = \frac{P_o}{R_o} \frac{dy}{dx}$$

$$\frac{P_o}{R_o} \frac{dy}{dx} = \frac{P_o}{R_o} \frac{y}{x} \left[\frac{w^2 x^2 - \frac{1.2\rho_o V_o^2}{P_o}}{\frac{1.2\rho_o V_o^2}{m P_o} - x^2 y^{(1/m)+1}}\right]$$

Now, the local sonic velocity of the air is given by the expression

$$C^2 = \left(\frac{dP}{d\rho}\right)_s$$

For an isentropic process

$$\frac{P}{\rho^k} = \text{const}$$

$$\ln P - k \ln \rho = \ln (\text{const})$$

$$\frac{dP}{P} - k \frac{d\rho}{\rho} = 0$$

$$\left(\frac{dP}{d\rho}\right)_s = k \frac{P}{\rho} = C^2$$

$$C_o^2 = k \frac{P_o}{\rho_o}$$

Therefore

$$\frac{dy}{dx} = \frac{y}{x} \left[\frac{w^2x^2 - 1.2k \dfrac{V_o^2}{C_o^2}}{1.2 \dfrac{k}{m} M_o^2 - x^2 y^{Ym+1}} \right]$$

$$\text{but} \quad \frac{V_o}{C_o} = M_o$$

$$\frac{dy}{dx} = \frac{y}{x} \left[\frac{w^2x^2 - 1.2kM_o^2}{1.2 \dfrac{k}{m} M_o^2 - x^2 y^{(1/m)+1}} \right]$$

Let

$$W^2 = x^2 y^{(1/m)+1}$$

$$y^{(1/m)+1} = \frac{W^2}{x^2}$$

$$y = \left(\frac{W}{x}\right)^{\frac{2}{(1/m)+1}} = W^{\frac{2}{(1/m)+1}} \; x^{-\frac{2}{(1/m)+1}}$$

$$\frac{dy}{dx} = x^{-\frac{2}{(1/m)+1}} \; \frac{2}{(1/m)+1} \; W^{\frac{2}{(1/m)+1}-1} \frac{dW}{dx} - W^{\frac{2}{(1/m)+1}} \; \frac{2}{(1/m)+1} \; x^{-\frac{2}{(1/m)+1}-1}$$

$$\frac{dy}{dx} = \frac{2m}{1+m}\left[\left(\frac{W}{x}\right)^{\frac{2m}{m+1}} W^{-1}\frac{dW}{dx} - \left(\frac{W}{x}\right)^{\frac{2m}{m+1}} x^{-1}\right] = \left(\frac{W}{x}\right)^{\frac{2m}{m+1}} .x^{-1}\left[\frac{w^2x^2-1.2kM_o^2}{1.2\frac{k}{m}M_o^2-x^2y^{(1/m)+1}}\right]$$

$$W^{-1}\frac{dW}{dx} - x^{-1} = \frac{m+1}{2m} .x^{-1}\left[\frac{w^2x^2 - 1.2k\,M_o^2}{1.2\frac{k}{m}M_o^2 - x^2y^{(1/m)+1}}\right]$$

$$W^{-1}\frac{dW}{dx} = x^{-1}\left[1 + \frac{m+1}{2m}\left(\frac{w^2x^2 - 1.2\,k\,M_o^2}{1.2\frac{k}{m}M_o^2 - x^2y^{(1/m)+1}}\right)\right]$$

$$\frac{dW}{dx} = \frac{W}{x}\left[1 + \frac{m+1}{2m}\left(\frac{w^2x^2 - 1.2k\,M_o^2}{1.2\frac{k}{m}M_o^2 - W^2}\right)\right]$$

$$\frac{dW}{dx} = \frac{W}{x}\left[\frac{1.2\,\frac{k}{m}\,M_o^2 - W^2 + \frac{m+1}{2m}\,w^2x^2 - \frac{1.2}{2}\,\frac{k}{m}\,M_o^2\,(M+1)}{1.2\,\frac{k}{m}\,M_o^2 - W^2}\right]$$

$$\frac{dW}{dx} = \frac{W}{x}\left[\frac{W^2 - \frac{m+1}{2m}\,w^2x^2 + \frac{1.2}{2}(m-1)\,\frac{k}{m}\,M_o^2}{W^2 - 1.2\,\frac{k}{m}\,M_o^2}\right]$$

For the Isentropic Case, m = k

$$\frac{dW}{dx} = \frac{W}{x}\left[\frac{W^2 - \frac{k+1}{2k}\,w^2x^2 + 1.2\,(k-1)\,M_o^2}{W^2 - 1.2\,M_o^2}\right] \qquad (6)$$

For the Isothermal Case m = 1

$$\frac{dW}{dx} = \frac{W}{x}\left[\frac{W^2 - w^2x^2}{W^2 - 1.2kM_o^2}\right]^* \qquad (7)$$

---

*Ecoulement Radial d'un Fluide Visqueux Entre deux Bisques Tres Rapproches Theorie du Palier a Air, E.A. Deuker et H. Wojetch, Revue Generale de l'Hydraulique, Vol. 17, 1951, pp 228-234, 285-294. This analysis parallels very closely that done in the reference cited.

There are three types of flow patterns which might exist in the bearing.

(a) The flow may be subsonic throughout the bearing.

(b) The flow may be sonic at R_o but subsonic at R

(c) The flow may be supersonic throughout the bearing.

In any event the flow is probably isentropic from the point where the gas leaves the capillary to the inner radius of the step bearing and probably beyond. The flow is also probably isothermal from the outer radius of the step inwards toward the inner radius.

These two types of flow are assumed, since it is expected that the expansion from the capillary end to the inner radius of the bearing (R_o) takes place over such a short distance that frictional effects are negligible. Since, however, the distance $R-R_o >>> h$, frictional effects must be important in the volume between the bearing surfaces. The flow is then other than adiabatic and probably isothermal.

If the flow is sonic at R_o and subsonic at R, two possibilities exist. The flow may become subsonic immediately after R_o or it may become supersonic. If the flow becomes supersonic, it will suffer a compression shock, with an attendant rise in pressure, and will then expand isothermally to the atmosphere. The pressure at R_o will be critical.

In the latter case, wherein the flow is supersonic throughout, the pressure at R_o will be critical. The pressure will then be reduced continuously across the bearing. It is possible, in this case, for the exit pressure at R to be something other than atmospheric. In fact it is entirely possible for the exit pressure (P_e) to be below atmospheric pressure (P_a). Since this condition is not favorable for high load carrying capacity, it is not a desirable state of affairs.

Another way of expressing the differential equation (7) is as follows:

$$\frac{dP}{dr} = \frac{(r^2 - AB)P}{Br(A-(Pr)^2)} \qquad (8)$$

where

$$A = \frac{1.2G^2RT}{4\pi^2h^2}$$

$$B = \frac{\pi h^3}{6\mu RTG}$$

$$AB = \frac{1.2Gh}{24\mu\pi g}$$

If P^2r^2 is much greater than A

then

$$\frac{dP}{dr} = \frac{(r^2-AB)P}{Br(-P^2r^2)} = -\frac{P(r^2-AB)}{P^2Br^3}$$

$$PdP = \frac{1}{B}\frac{(Ab-r^2)}{r^3}dr = \frac{A}{r^3}dr - \frac{1}{Br}dr$$

$$\frac{P^2}{2} = -\frac{A}{2r^2} - \frac{\ln r}{B} + C \quad \text{where } r = R_o,\ P = P_o$$

$$\frac{P_o^2}{2} = -\frac{A}{2R_o^2} - \frac{\ln R_o}{B} + C$$

$$C = \frac{P_o^2}{2} + \frac{A}{2R_o^2} + \frac{\ln R_o}{B}$$

$$\frac{P^2}{2} - \frac{P_o^2}{2} = \frac{A}{2}\left(\frac{1}{R_o^2} + \frac{1}{r^2}\right) + \frac{1}{B}\ln\frac{R_o}{r}$$

$$P_o^2 - P^2 = A\left(\frac{1}{r^2} - \frac{1}{R_o^2}\right) - \frac{2}{B}\ln\frac{R_o}{r}$$

$$P_o^2 - P^2 = \frac{1.2G^2RT}{4\pi^2h^2}\left(\frac{1}{r^2} - \frac{1}{R_o^2}\right) - \frac{12\mu RTG}{\pi h^3}\ln\frac{R_o}{r}$$

$$\frac{\pi h^3(P_o^2 - P^2)}{12\mu RT} = \frac{1.2G^2h}{48\mu\pi}\left(\frac{1}{r^2} - \frac{1}{R_o^2}\right) + G\ln\frac{r}{R_o}$$

Therefore

$$G = \frac{\pi h^3(P_o^2 - P^2)}{12\mu RT\ln\frac{r}{R_o}} - \frac{1.2G^2h}{48\pi\mu\ln\frac{r}{R_o}}\left(\frac{1}{r^2} - \frac{1}{R_o^2}\right) \tag{9}$$

when $r = R_a, P = P_a$

Therefore

$$G = \frac{\pi h^3\ (P_o^2 - P_a^2)}{12\mu RT\ln\frac{Ra}{R_o}} - \frac{1.2G^2h}{48\mu\ln\frac{Ra}{R_o}}\left(\frac{1}{R^2} - \frac{1}{R_o^2}\right) \tag{10}$$

If $AB <<< r^2$

$$\frac{dP}{dr} = \frac{Pr}{AB-BP^2r^2}$$

$$\frac{dr}{dP} = \frac{AB-BP^2r^2}{Pr} = \frac{AB}{Pr} - BPr$$

$$\frac{dr}{dP} = \frac{AB}{Pr} - BPr$$

$$\text{Let} \quad \mu = P^2r^2$$

$$\sqrt{\mu} = Pr$$

$$r = \frac{\mu}{P}^{1/2}$$

$$\frac{dr}{dP} = -\frac{\mu^{1/2}}{P^2} + \frac{1}{P}\cdot\frac{1}{2\mu^{1/2}}\frac{d\mu}{dP} = \frac{1}{2P\sqrt{\mu}}\frac{d\mu}{dP} - \frac{\mu^{1/2}}{P^2}$$

Therefore

$$\frac{1}{2P\mu^{1/2}}\frac{d\mu}{dP} - \frac{\mu^{1/2}}{P^2} = \frac{AB}{\mu^{1/2}} - B\mu^{1/2}$$

$$\frac{d\mu}{dP} = \frac{2\mu}{P} + 2ABP - 2PB\mu = \left(\frac{2}{P} - 2BP\right)\mu + 2PAB$$

$$\frac{d\mu}{dP} + 2\left(BP - \frac{1}{P}\right)\mu = 2ABP$$

This equation is linear and an integrating factor is

$$e^{\int 2(BP-\frac{1}{P})dP} = e^{BP^2 - \ln P^2} = \frac{e^{BP^2}}{P^2}$$

$$\mu \frac{e^{BP^2}}{P^2} = 2AB \int P \frac{e^{BP^2}}{P^2} dP + c$$

$$\frac{\mu}{P^2} = \frac{2AB}{e^{BP^2}} \int \frac{e^{BP^2}}{P} dP + \frac{C}{e^{BP^2}}$$

$$\text{Let} \quad BP^2 = x$$

$$P = \left(\frac{x}{B}\right)^{1/2}$$

$$dP = \frac{1}{2B^{1/2}} \frac{dx}{x^{1/2}}$$

Therefore

$$\int \frac{e^{BP^2}}{P} dP = \int \frac{e^x}{\left(\frac{x}{B}\right)^{1/2}} \cdot \frac{1 \, dx}{2(Bx)^{1/2}} = \int \frac{e^x dx}{2x}$$

$$\int \frac{e^x dx}{2x} = \frac{1}{2}\left[\ln x + x + \frac{x^2}{2 \cdot 2!} + \frac{x^3}{3 \cdot 3!} + \frac{x^4}{4 \cdot 4!} + \cdots\cdots + C_1\right]$$

Therefore

$$\frac{\mu}{P^2} = r^2 = \frac{AB}{e^{BP^2}}\left[\ln BP^2 + BP^2 + \frac{B^2P^4}{2 \cdot 2!} + \frac{B^3P^6}{3 \cdot 3!} + \cdots\right] + \frac{C}{e^{BP^2}}$$

$$\text{when} \quad r = R_o, \quad P = P_o$$

$$r^2 = R_o^2 \, e^{B(P_o^2 - P^2)} - \frac{AB}{e^{BP^2}}\left[\ln \frac{P_o^2}{P^2} + B\,(P_o^2 - P^2) + \frac{B^2(P_o^4 - P^4)}{2 \cdot 2!} + \cdots\right] \quad (11)$$

there is a limited amount of experimental data to test the applicability of equations (5), (10), and (11). Licht and Fuller* presented some data for a hydrostatic step bearing with the following specifications:

$R_o = 1$ in., $R_a = 3$ in., $\mu = 2.61 \times 10^{-9} \frac{\text{lb-sec}}{\text{in.}^2}$, $h = 0.0015$ in.

$P_o = 44.7$ psia, $P_a = 14.7$ psi

When using the equations for incompressible flow, a flow of 1110 cubic in./min is calculated. Equation (5) gives a flow of 2244 cubic in./min. Equation (10) gives a flow of 2410 cubic in./min. The actual flow measured 2440 cubic in./min. The use of equation (10) reduces the error of equation (5) from about 7% to about 1 1/4%.

* Paper No. 54-LuB-18, ASME. Presented at the ASME-ASLE Lubrication Conference, Baltimore, Maryland, October 18-20, 1954.

Equations (5), (9), and (11) relate the pressure in the bearing to its location. In order to obtain the resultant pressure force or load carrying capacity of the bearing, these equations have to be integrated across the bearing. Because of the complexity of the pressure equations, the integration is best carried out by the use of numerical methods.

One more equation must be derived before we can begin an analysis of a gas lubricated bearing. This is the equation which tells us the magnitude of the pressure drop between the exit of the capillary and the inner radius (R_o) of the bearing

The energy equation relating these points is given in equation (12) below:

$$\frac{V_c^2-V_o^2}{2} = \left(\frac{k}{k-1}\right) \frac{P_o}{\rho_o} \left[1-\left(\frac{P_c}{P_o}\right)^{\frac{k-1}{k}}\right] \tag{12}$$

If the velocity V_c is much less than V_o, and in most cases it will indeed be so, then equation (12) can be rearranged to give

$$\frac{P_c}{P_o} = \left[1 + \frac{k-1}{2k} \frac{V_o^2}{P_o/\rho_o}\right]^{\frac{k}{k-1}}$$

$$\text{but} \quad \frac{kP_o}{\rho_o} = C_o^2$$

Therefore

$$\frac{P_c}{P_o} = \left[1 + \frac{k-1}{2} \frac{V_o^2}{C_o^2}\right]^{\frac{k}{k-1}} \tag{13}$$

$$\text{but} \quad \frac{V_o}{C_o} = M_o$$

Therefore

$$\frac{P_c}{P_o} = \left(1 + \frac{k-1}{2} M_o^2\right)^{\frac{k}{k-1}} \tag{14}$$

For k = 1.4

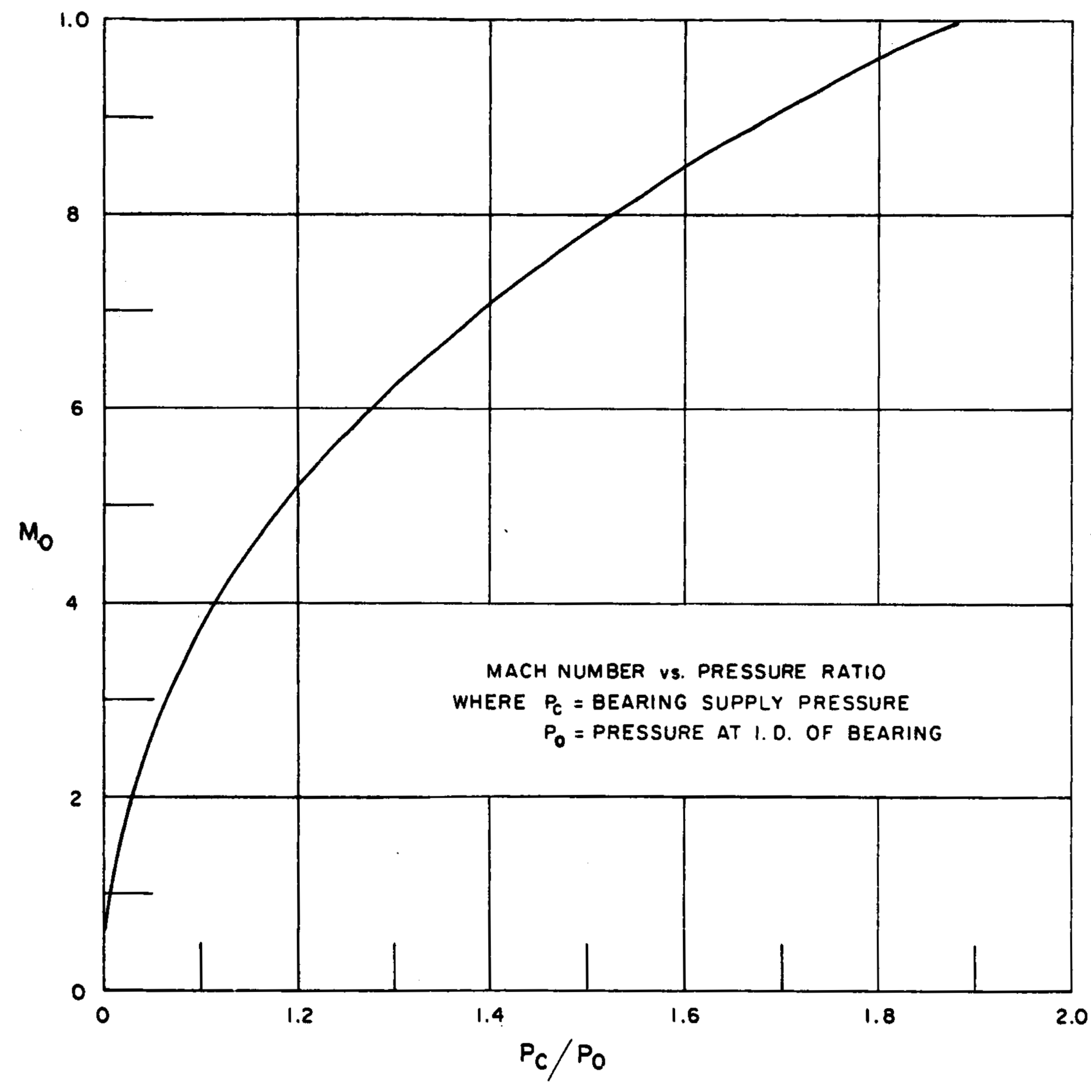
$$\frac{P_c}{P_o} = \left(1 + .2M_o^2\right)^{3.5} \tag{15}$$

Figure 7 is a plot of this equation.

Before we attempt to use any of the equations we have derived, it would be well to discuss the preliminary considerations and our method of attack.

Three possible types of flow are assumed to exist:

(a) The flow is subsonic throughout the bearing.
(b) The flow is supersonic throughout the bearing.
(c) The flow is supersonic from the inner radius of the bearing to some point between the inner and outer radii of the bearing and subsonic from there to the outer radius. A compression shock joins the two types of flow. In cases (b) and (c), the flow is sonic at the inner radius and all the required characteristics of the gas are known at this point. The pressure distribution may be found in the following manner.



REPORT F-A1914

FIGURE 7

(1) Assume a value of pressure at the exit to the Capillary. The flow in the capillary is assumed to be isothermal so that the density of the gas at this point can be computed.

(2) Compute the value of the density of the gas at the exit to the capillary.

(3) Since flow is sonic at R_o, the pressure at this point is critical. Figure 7 shows that $\frac{P_c}{P_o} = 1.895$. Compute P_o.

(4) Compute ρ_o from the equation $\rho_o = \rho_c \left(\frac{P_o}{P_c}\right)^{1/k}$,

since this flow is isentropic.

(5) Compute V_o from the equation

$$V_o = C_o = \sqrt{\frac{kP_o}{\rho_o}}$$

(6) Calculate $G = \rho_o A_o V_o$.

(7) Use equation (4) to check on the assumed value of P_c. P_s is of course known. If the calculated value of P_c does not check with the assumed value repeat steps (1) through (7).

(8) Compute $A = \frac{1.2G^2RT}{4\pi^2h^2g}$, $B = \frac{\pi h^3}{6\mu RTG}$.

In this case assume that the temperature is equal to the ambient exit temperature.

(9) Using the differential pressure equation for isothermal flow

(Equation (8)), $\left[\frac{dP}{dr} = \frac{(r^2-AB)P}{Br(A-P^2r^2)}\right]$, integrate numerically from

the outer radius towards the inner radius until the expression $(A-P^2r^2)$ vanishes.

(10) Using the same differential equation, integrate numerically from the inner radius toward the outer radius until the radius at which the denominator $Br(A-P^2r^2)$ vanished in step (9).

(11) Join the pressure vs. radius curve with a vertical line at this radius as shown below in Figure 8.

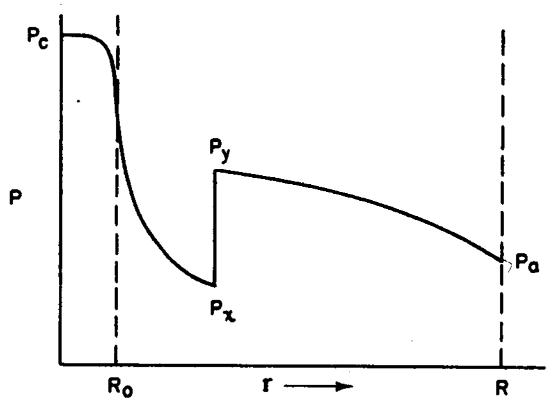


FIGURE 8

(12) Since it is known that a compression shock occurs over a finite distance, it would probably be of interest to attempt to describe this phenomenon.

(13) The value P_y (after shock) is known from step (9).

(14) Using the expression $\rho_y = \rho_a \frac{P_y}{P_a}$, compute ρ_y.

(15) Compute V_y from the expression $V_y = \frac{G}{2\pi r_y \rho_y h}$.

(16) Compute C_y from the equation $C_y = \sqrt{k \frac{P_y}{\rho_y}}$.

(17) $M_y = \frac{V_y}{C_y}$.

(18) From Keenan and Keyes' "Gas Tables," find $\frac{P_x}{P_y}$, solve for P_x subscript x refers to before shock conditions.

(19) Find the value of P_x on the curve obtained in step (10).

(20) Join this point to P_y on the pressure radius curve as shown by the dotted line in Figure 9, below.

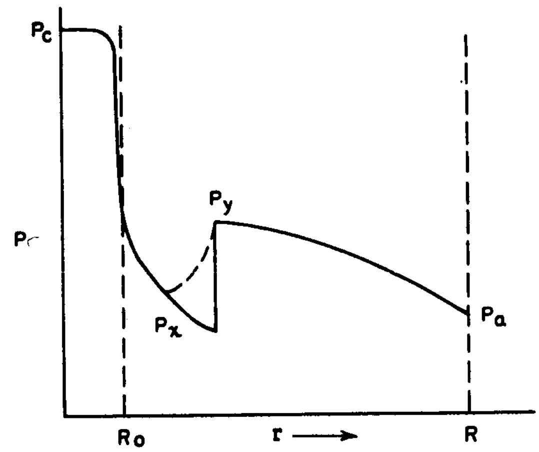


FIGURE 9

If the flow is supersonic throughout the bearing, only step (10) is required to determine the pressure distribution. The exit pressure is now probably other than the ambient exit pressure.

If the flow is subsonic throughout, then only step (10) is required to determine the pressure distribution. Now, however, we have the problem of determining the mass flow and the velocity at the inner radius of the bearing.

To determine if sonic or subsonic flow occurs in the bearing, the following steps are taken.

(21) For a first approximation of the flow available use equation (5)

$$G = \frac{\pi h^3(P_o^2-P_a^2)}{12\mu RT \ln\frac{R}{R_o}}, \text{ where } P_o \text{ is critical.}$$

(22) Compute $V_o = \frac{G}{2\pi R_o h\rho_o}$

(23) Compute the velocity of sound according to the expression

$$C = \sqrt{k\frac{P_o}{\rho_o}} \qquad \text{where } \rho_o = \rho_a \frac{P_o}{P_a}$$

(24) Compute the Mach number. $M_o = \frac{V_o}{C_o}$

If this number is equal to or greater than 1, then the flow is sonic at the inner radius of the bearing. If it is less than 1, the flow is subsonic.

Because the pressure vs. radius curve is less steep when subsonic flow prevails than when supersonic flow prevails, it is desirable to design the bearings for subsonic flow.

The main problem in air bearing design becomes therefore the determination of the flow at subsonic conditions; or the pressure at the inner radius of the bearing.

For a first approximation (in subsonic flow) start with Equation (5)

$$G = \frac{\pi h^3(P_o^2-P_a^2)}{12\mu RT\ell nR_a} \qquad (5)$$

but

$$G = 2\pi R_o h\rho_o M_o \sqrt{k\frac{P_a}{\rho_a}} \quad \text{where } M \gtrless 1$$

Then

$$\rho_o = \frac{\pi h^3(P_o^2 - P_a^2)}{\left[12\mu RT \ln \frac{R}{R_o}\right]\left[2\pi R_o M_o \sqrt{k \frac{P_a}{P_a}}\right]}$$

$$\rho_o = \frac{h^2(P_o^2 - P_a^2)}{24\, R_o \mu RT \sqrt{k \frac{P_a}{\rho_a}} \ln \frac{Ra}{R_o} M_o}$$

$$\rho_o = \frac{P_o}{P_a} \cdot \rho_a = \frac{h^2(P_o^2 - P_a^2)}{24 R_o \mu RT \ln \frac{Ra}{R_o} M_o \sqrt{k \frac{P_a}{P_a}}}$$

$$P_o = \frac{P_a h^2(P_o^2 - P_a^2)}{24\mu R_o \rho_a RT \ln \frac{Ra}{R_o} M_o \sqrt{k \frac{P_a}{\rho_a}}}$$

$$P_o^2 - P_o \quad \frac{24\mu R_o RT \ln \frac{Ra}{R_o} M_o \sqrt{k \frac{\rho_a}{P_a}}}{h^2} \quad - P_a^2 = 0$$

If we solve for P_o with M=1, we can find the value of pressure required for critical flow to occur at the throat of the bearing. Let us consider the following example

Given:

$R_o = 1 \quad h = 1.5x10^{-3} \qquad \mu = 2.61x10^{-9}\ \frac{\text{lb-sec}}{\text{in}^2} \qquad R = 53.3\ \frac{\text{ft. lb}}{\text{lb}\ ^{\circ}\text{R}}$

$R = 3 \quad P_a = 14.7\ \text{psia} \qquad \rho_a = 0.002378\ \text{slugs/ft}^3 \quad T = 520^{\circ}\text{R}$

$M = 1$

$$\ln \frac{R}{R_o} = \ln 3 = 1.1$$

$$\sqrt{k\frac{\rho_a}{P_a}} = \sqrt{\frac{1.4x0.002378}{14.7x144}} = \sqrt{1.57 \times 10^{-6}} = 1.25x10^{-3}\ \frac{\text{sec}}{\text{ft}} = 1.04x10^{-4}\ \frac{\text{sec}}{\text{in.}}$$

$$\frac{24\ R_o \mu R T \ln \frac{R_a}{R_o}\ M \sqrt{k\frac{P_e}{P_e}}}{h^2} = \frac{24x1x2.61x10^{-9}x53.3x12x386x520x1.1x1.04x10^{-4}}{2.25 \times 10^{-6}}$$

$$= \frac{24x2.61x53.3x12x386x520x1.1x1.04x10^{-7}}{2.25}$$

$$= 410\ \text{psi}$$

Therefore

$$P_o^2 - 410\ P_o - 216 = 0$$

$$P_o = 410\ \text{psi}$$

In order for Mach 1 to be obtained at the inner radius of this step bearing, the pressure there will have to be 410 psi.

If we wish to determine the relationship between P_o and M_o we use the expression

$$P_o^2 - 410.5\, M_o P_o - 216 = 0$$

for $M_o = 0.5$ $\quad P_o = 207$

$M_o = 0.3$ $\quad P_o = 124.5$

$M_o = 0.2$ $\quad P_o = 84.5$

$M_o = 0.1$ $\quad P_o = 45$

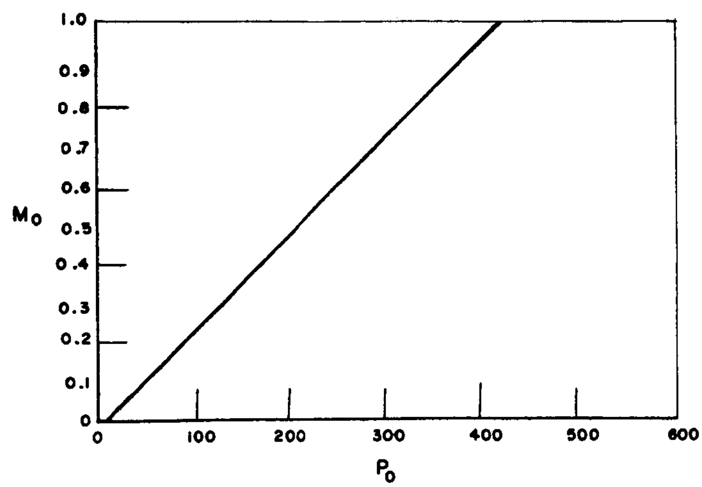


FIGURE 10

Figure 10 is a plot of M_o vs. P_o for this problem

There are now two equations which can be used to get us started in the design of a gas bearing. They are:

$$\text{(a)} \quad P_o^2 - P_o\left[\frac{24\mu R_o RT \ln R_a/R_o \; M_o\sqrt{k\,\rho_a/P_a}}{h^2}\right] - P_a^2 = 0 \qquad (16)$$

$$\text{(b)} \qquad \frac{P_c}{P_o} = \left(1 + \frac{k-1}{2} M_o^2\right)^{k/k-1} \qquad (14)$$

The first step in determining the load carrying capacity of a gas-lubricated hydrostatic bearing operating under subsonic conditions is to plot equation (16) as was done in Figure 10. To determine the correct value of P_o, we select a value of M_o between 0 and 1 and find the corresponding value of P_o from this plot. Using this value of P_o and the known value of P_c, we compute P_c/P_o. Using equation (14), we compute a value of M_o and check this value of M_o with the originally assumed value. We repeat this procedure until the computed value of M_o agrees with the assumed value.

Let us consider a typical problem:

$P = 100$ psia	$R = 53.3 \times 12 \times 386$ in.^{2}/°Rsec^2
$P_a = 14.7$ psia	$T = 520$°R
$R_o = 1/4$ in.	$\mu = 2.61 \times 10^{-9}$
$h = 1.5 \times 10^{-3}$ in.	$R_a = 2$

Therefore $\quad \frac{R}{R_o} = \frac{2}{1/4} = 8, \; \ln\frac{R}{R_o} = \ln 8 = 2.08$

$$24\mu R_o RT \ln\frac{R_a}{R_o} M_o \sqrt{\frac{k\rho_a}{P_a}} = \frac{24 \times 2.61 \times 10^{-9} \times \frac{1}{4} \times 53.3 \times 12 \times 386 \times 520 \times 7.08 \times M \times 1.04 \times 10^{-4}}{2.25 \times 10^{-6}}$$

$$= 193 M_o \text{ psi}$$

Therefore $$P_o^2 - 193MP_o - P_a^2 = 0$$

M_o	P_o
1	194 psi
0.5	98.5 psi
0.3	61.5 psi
0.2	43.5 psi
0.1	27 psi
0	14.7 psi

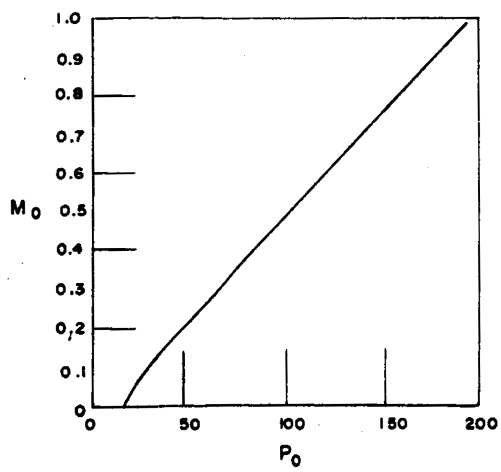


FIGURE 11

Figure 11 is a plot of this data.

Let us select as a first choice a Mach number of 0.3. Figure 11 shows that a Mach number (M_o) of 0.3 calls for a value of 61.5 psi. Figure 7 shows that P_c/P_o at $M_o = 0.3$ is 1.065. Therefore P_c would be $61.5 \times 1.065 = 65.5$ psi; obviously our first estimate was too low. Let us let $M_o = 0.4$. Figure 11 gives $P_o = 82$. Figure 7 gives $P_c/P_o = 1.115$. Therefore $P_c = 91.5$. Try $M_o = 0.43$. Figure 11 gives $P_o = 89$. Figure 7 gives $P_c/P_o = 1.135$. Therefore $P_c = 101$ psi. Therefore $M_o = 0.43$.

Now we can compute G from the expression $G = \rho_o \cdot 2\pi R_o h V_o$. If a high degree of accuracy is required, this value of M_o can be adjusted again by trial and error matching of equations (8) and (14). When this is done the correct value of G will have been determined. If equation (8) is used in this manner, one of the benefits which will accrue will be a pressure distribution in the bearing. If equation (8) is not used (less accuracy permissible), the pressure distribution in the bearing can be obtained with the use of equations (5) or (9). From the pressure distribution, the resultant pressure force or load carrying capacity can be computed.

Let us discuss equation (16) for a moment. Suppose we rewrite it as follows:

$$P_o - KM_o \frac{R_o}{h^2} P_o - P_a^2 = 0$$

where K is $24\mu RT \ln \frac{R_a}{R_o} \sqrt{k\rho_a/P_a}$. Therefore,

$$P_o = \frac{KM_o \frac{R_o}{h^2} + \sqrt{K^2 M_o^2 \frac{R_o^2}{h^4} + 4P_a^2}}{2}$$

If we set M=1, then P_o becomes the pressure at the inside diameter required before the flow at that point becomes sonic.

Therefore
$$P_o = K \frac{R_o}{h^2} + \sqrt{K^2 \frac{R_o^2}{h^4} + 4P_a^2} \tag{17}$$

If we now visualize an experiment involving a circular step bearing with controlled film thickness and constant pressure P_o, we can understand the effect of film thickness on the pressure distribution and load-carrying capacity of the bearing. As we increase the film thickness, equation (17) tells us that we need less pressure at R_o for critical flow to occur there. Another way of interpreting this phenomenon is as follows: as the film thickness is increased, the Mach number of the flow (M_o) at the inside diameter of the step bearing increases for a constant value of P_o until critical flow is reached.

One thing must be remembered before the above method can be used. The method outlined above is good only when the flow governing area is determined by the expression $2\pi R_o h$. Consider Figure 12, below.

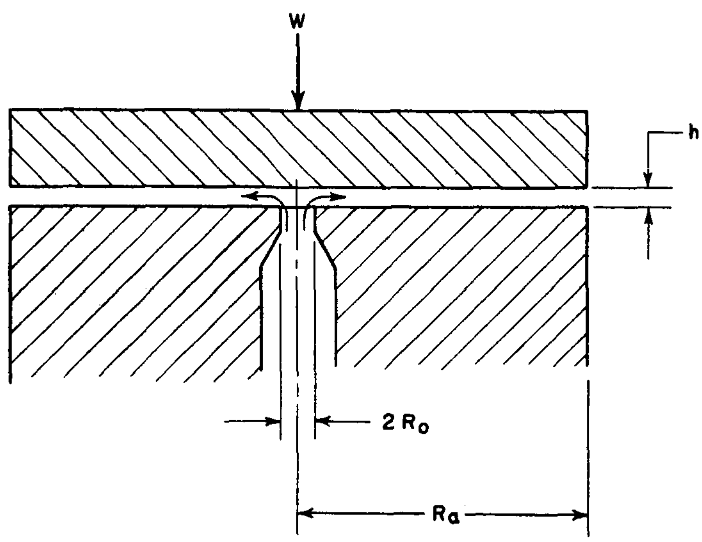


FIGURE 12

The flow of air first passes through the orifice area πR_o^2 and then through a second orifice area $2\pi R_o h$. For a configuration similar to that shown in Figure 12, the latter orifice is the flow governing area, since $R_o \gg 2h$. However, when we add a recess in the bearing as shown in Figure 13. The two orifices which compete to govern the flow are πa^2 and $2\pi R_o h$. If $2\pi R_o h$ is larger than πa^2, then the latter becomes the flow control area. In this instance a relatively large pressure drop will occur through the orifice πa^2 and a relatively small drop through the orifice $2\pi R_o h$.

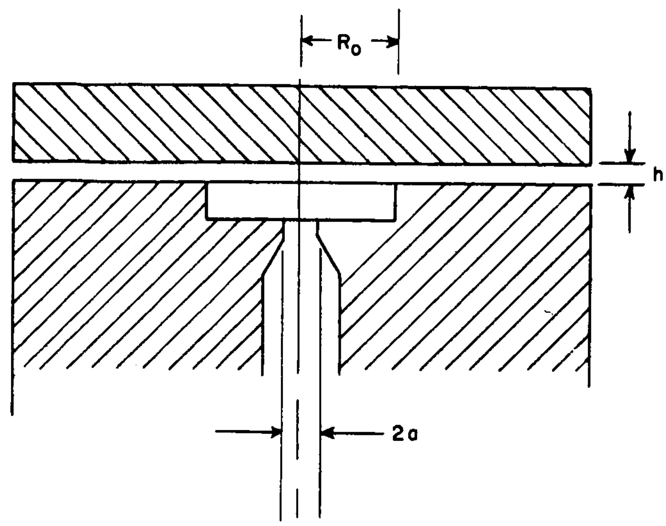


FIGURE 13

Equations (16) and (14) can still be used with the understanding that the Mach numbers are no longer equal. In equation (16) the Mach number (M_o) is still the Mach number at R_o but the Mach number in equation (14) is the Mach number at the exit of the orifice defined by the area πa^2.

The two equations are then

$$P_o^2 - P_o \left[\frac{24 \mu R_o RT \quad n\, R/R_o \; M_o \sqrt{k \rho_a / P_a}}{h^2} \right] - P_a^2 = 0 \qquad (16)$$

$$\frac{P_c}{P_o} = \left(1 + \frac{k-1}{2} M^2\right)^{k/k-1} \qquad (14a)$$

In equation (14a) P_c is the pressure before the orifice and P_o that after it. The entire recess area πR_o^2 is then assumed to be at pressure P_o.

The two equations are related in the following manner:

(1) Select a value for M_o in equation (16) and compute P_o.

(2) Compute C_o from $C_o = \sqrt{k\ P_o/\rho_o}$ where $\rho_o = \rho_a\ P_o/P_a$.

(3) Compute V_o from $V_o = C_o M_o$.

(4) Compute G from $G = 2\pi R_o h \rho_o V_o$.

(5) Compute V_c (the velocity at the exit to the orifice πa^2) from $V_c = G/\pi a^2 \rho_o$.

(6) Compute M from V_c/C_o.

(7) Compute P_c/P_o from equation (14a).

(8) Compute P_c and check with given value.

(9) If the P_c obtained in step (8) is not equal to the given value, repeat steps (1) through (8) until they do.

When the cross sectional areas of the two orifices in question approach each other or equal each other in magnitude, we have a situation which is not yet completely understood and is better avoided.

V. LOADS ON A TYPICAL TURBOJET ENGINE

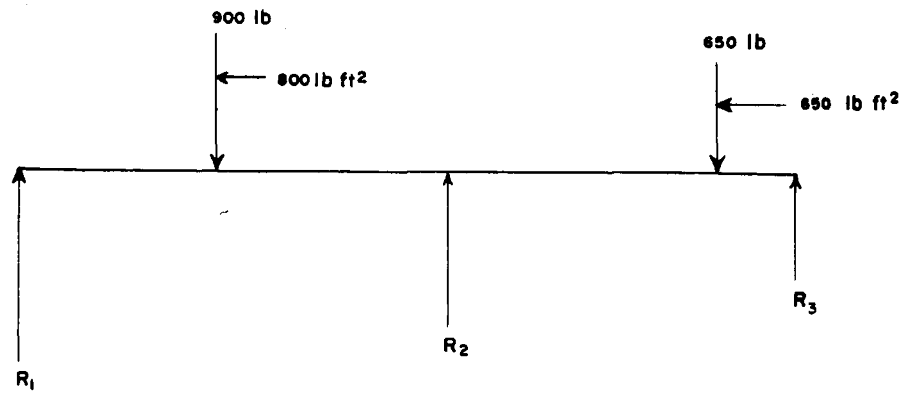


FIGURE 14

Figure 14 is a sketch indicating the static forces, moments of inertia, and dimensions of a typical engine shaft.

Using the usual equations describing the deflection, slope, and bending moment of the above beam one can solve for the bearing reactions R_1, R_2, and R_3. Because the calculations are long they are not presented here, but in the appendix to this report.

The static forces are:

$$R_1 = 414 \text{ lb}$$
$$R_2 = 795 \text{ lb}$$
$$R_3 = 341 \text{ lb}$$

For 4.0 rad/sec precessional velocity:

$$R_1 = 12{,}000 \text{ lb}$$
$$R_2 = 6{,}750 \text{ lb}$$
$$R_3 = 18{,}750 \text{ lb}$$

For 2.0 rad/sec precessional velocity:

$$R_1 = 6000 \text{ lb}$$
$$R_2 = 3375 \text{ lb}$$
$$R_3 = 9375 \text{ lb}$$

The maximum bearing load specified by section 3.15 of MIL-E-5007A is:

$$R_1 = 12{,}000 + 414 = 12{,}414 \text{ lb}$$
$$R_2 = 6{,}750 + 795 = 7{,}545 \text{ lb}$$
$$R_3 = 18{,}750 + 341 = 19{,}091 \text{ lb}$$

Other flight conditions specified in section 3.14 of the above specification are:

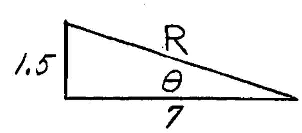
(1) 10g down and 1.5g side load:

$$R = \sqrt{100 + 2.25g} = \sqrt{102.25g} = 10.125g \qquad R_1 = 4200 \text{ lb}$$
$$\theta = \tan^{-1} 0.15 = 8.6° \qquad R_2 = 8060 \text{ lb}$$
$$R_3 = 3450 \text{ lb}$$

(2) 6g down + 4g side + 2 rad/sec pitch side load:

R_1 = 8500 lb ± 17.1° from horizontal, left or right
R_2 = 8120 lb ± 36.0° from horizontal, left or right
R_3 = 9000 lb ± 13.2° from horizontal, left or right

(3) 7g up + 1.5g side load:



$$R = \sqrt{49 + 2.25g} = \sqrt{51.25g} = 7.15g \qquad R_1 = 2950 \text{ lb}$$
$$\theta = \tan^{-1} 1.5/7 = 12.1° \qquad R_2 = 5660 \text{ lb}$$
$$R_3 = 2420 \text{ lb}$$

The following are the most important loading conditions for each radial bearing:

Bearing No. 1:

2950 lb ± 12.1° from the vertical, up
4200 lb ± 8.6° from the vertical, down
8500 lb ± 17.1° from the horizontal, left or right
12,414 lb ± 0° in the vertical direction, up or down

Bearing No. 2:

5660 lb ± 12.1° from the vertical, up
8060 lb ± 8.6° from the vertical, down
8120 lb ± 36.0° from the horizontal, left or right
7545 lb ± 0° in the vertical direction, up or down

Bearing No. 3:

2420 lb ± 12.1° from the vertical, up
8060 lb ± 8.6° from the vertical, down
9000 lb ± 13.2° from the horizontal, left or right
19,091 lb ± 0° in the vertical direction, up or down

These loads are shown schematically in Figures 15, 16 and 17.

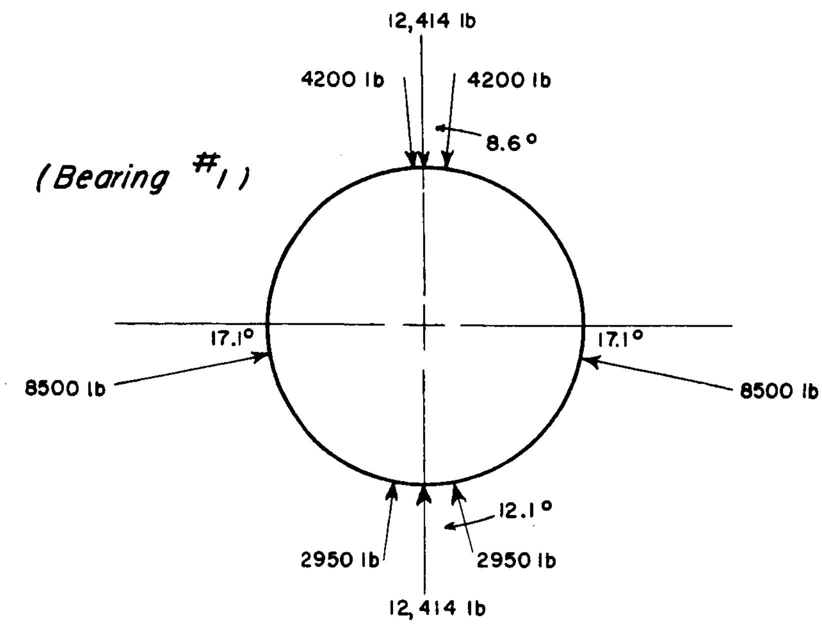


FIGURE 15

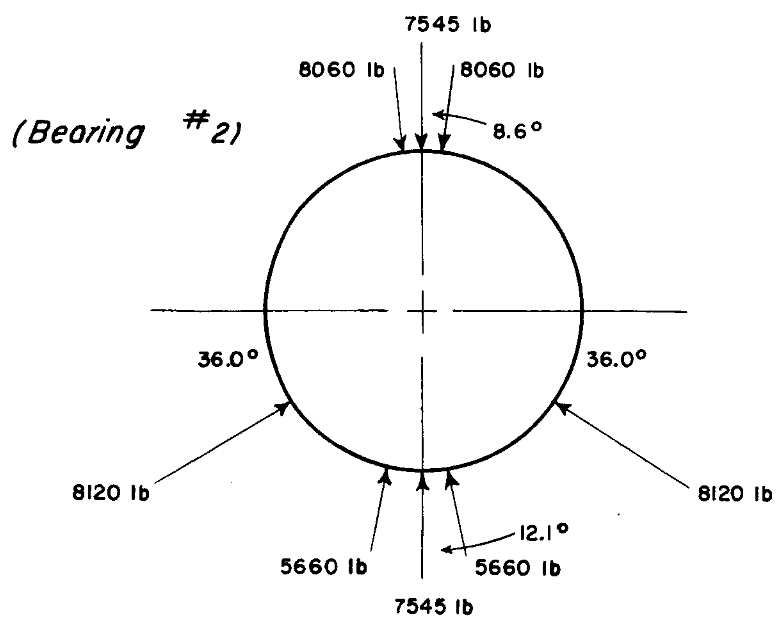


FIGURE 16

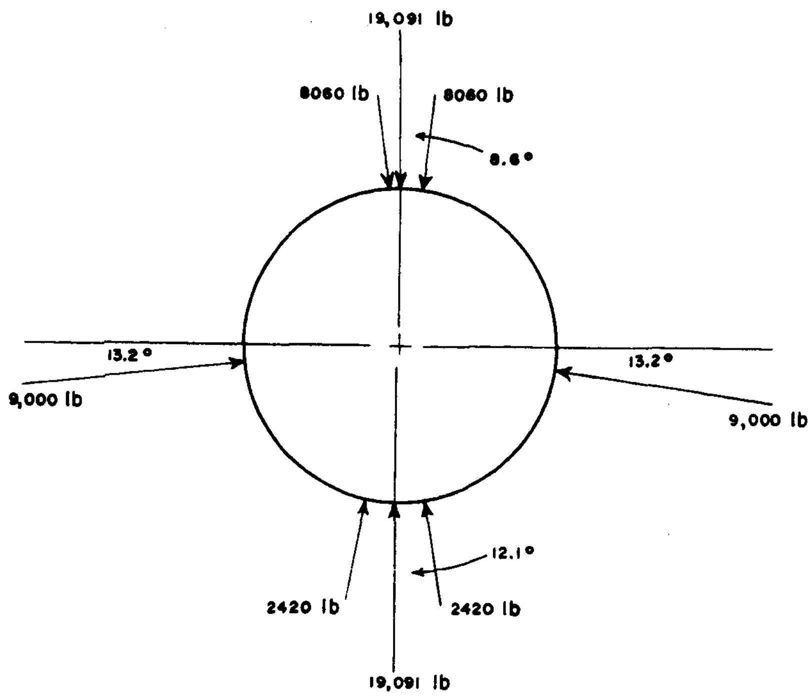


FIGURE 17

Let us consider a simple design for one of the bearings (Bearing No. 1) and run through a possible design. This should show if it is feasible to design gas bearings for this purpose at this time.

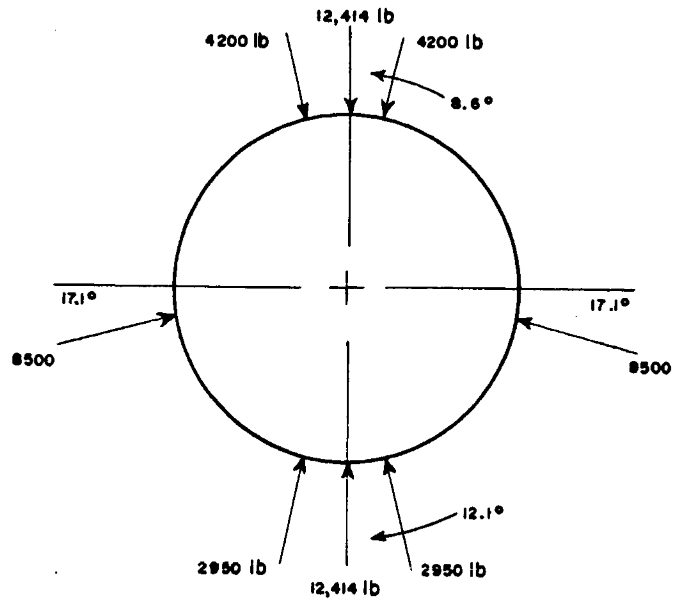


FIGURE 18

In order to preclude the possibility of whirl of the shaft we shall contemplate a tilting pad journal bearing with 4 pads each with slightly less than 90° included angle as shown in Figure 19.

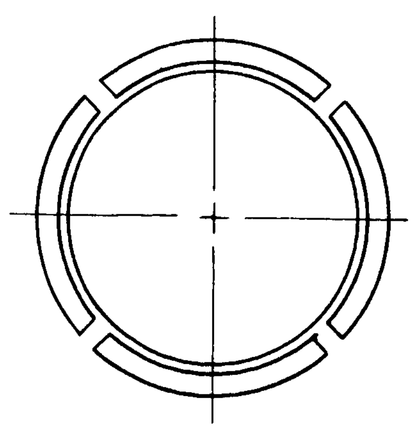


FIGURE 19

The diameter of the bearing is 14 inches and its length is 20 inches. The approximate projected dimensions of one pad is 10 in. x 20 in. Let us think of this bearing as composed of two circular step bearings each with its own entrance port as shown in Figure 20. The outside diameter of each step bearing will then be 10 in. Let the I.D. be 8 in. as shown in Figure 21.

The two equations which will be used to determine the feasibility of gas bearings for this application are:

$$\text{(a)}\quad P_o^2 - P_o\left[\frac{24\mu R_o RT\ \ell n\ R/R_o\ M_o\sqrt{k\rho_a/P_a}}{h^2}\right] - P_a^2 = 0 \qquad (16)$$

$$\text{(b)}\quad \frac{P_c}{P_o} = \left(1+\frac{k-1}{2}M^2\right)^{k/k-1} \qquad (14)\text{ or }(14a)$$

Let the diameter of the inlet orifice be 0.10 in.

$$a = 0.050 = 50 \times 10^{-3}$$

Now $2\pi R_o h = 6.28 \times 4 \times h = 25.1\ h$. h is or will be of the order of 1×10^{-3},

$$2\pi R_o h = 25.12 \times 10^{-3}$$

but $\pi a^2 = 3.17 \times (50 \times 10^{-3})^2 = 7850 \times 10^{-6} = 7.850 \times 10^{-3}$. $\pi a^2 < 2\pi R_o h$, so that the former orifice will probably govern the flow.

Let

$P_c = 125$ psia

$P_a = 14.7$ psia

$R = 53.3 \times 12 \times 386$ in.^{2}/°R sec^2

$\sqrt{k\rho_a/P_a} = 1.04 \times 10^{-4}$ sec/in.

$T = 520$°R

$\mu = 2.61 \times 10^{-9}$ lb-sec/in.^{2}

$R_o = 4$ in.

$R_a = 5$ in.

h_o = radial clearance = 0.35×10^{-3}

$\rho_a = .00236$ slug/ft^3

$k = 1.4$

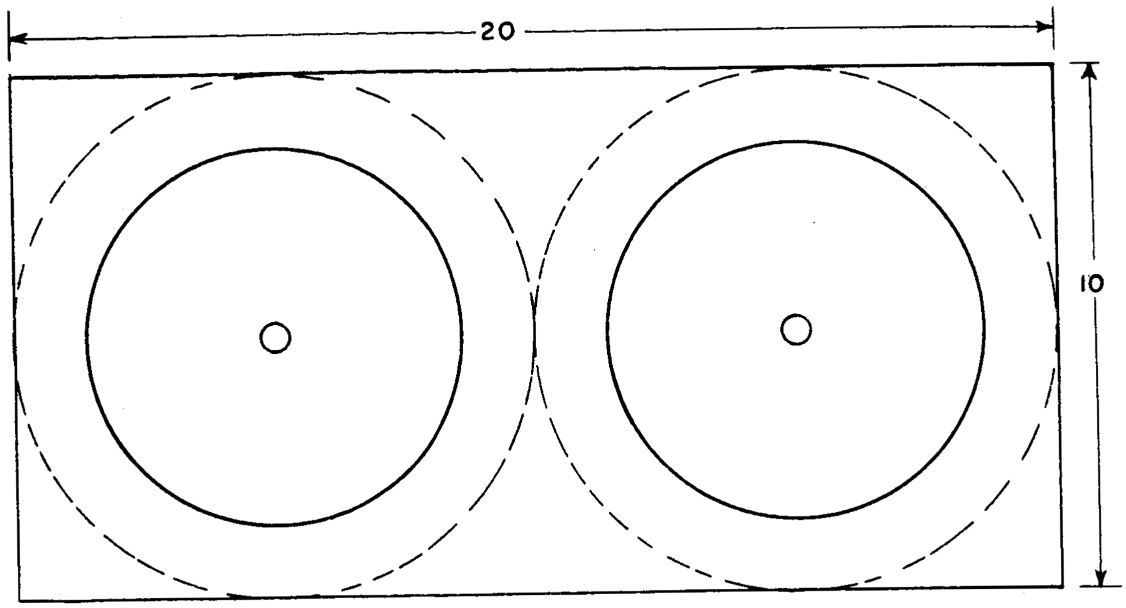


FIGURE 20

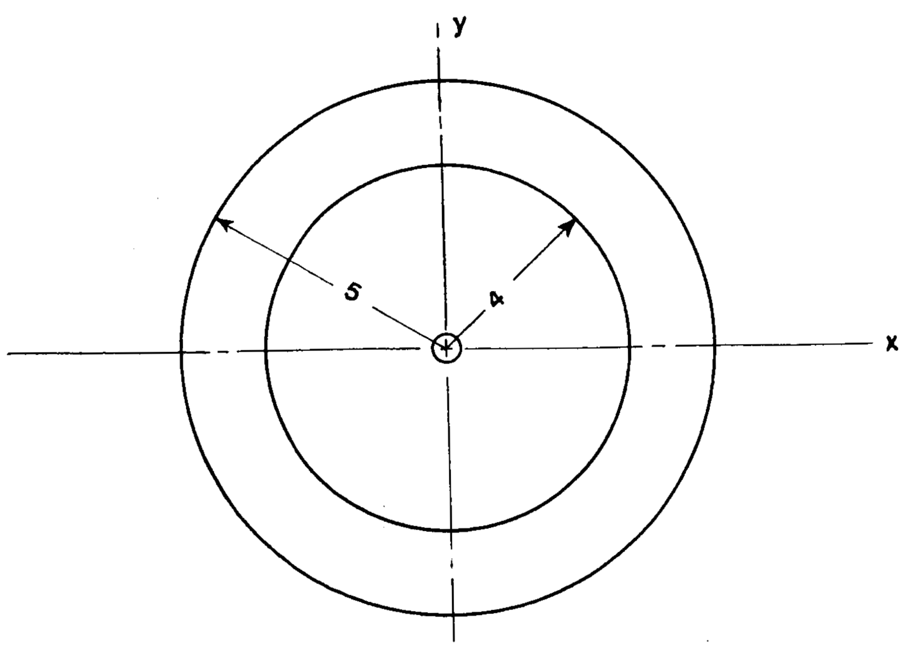


FIGURE 21

Therefore $$\frac{24\mu R_o RT \ln R_a/R_o \, M_o\sqrt{k\rho_a/P_a}}{h^2}$$

$$= \frac{24 \times 2.61 \times 10^{-9} \times 4 \times 53.3 \times 12 \times 386 \times 520 \times 0.223 \times 1.04 \times 10^{-4} M_o}{(3.5)^2 \times 10^{-6}}$$

$$= 61.2 \; M_o \text{ psi}$$

Therefore $$P_o^2 - 61.2 \; M_o P_o - P_a^2 = 0$$

If $h = 3.0 \times 10^{-3}$,

$$P_o^2 - 83.5 \; M_o P_o - P_a^2 = 0$$

If $h = 2.5 \times 10^{-3}$,

$$P_o^2 - 120 \; M_o P_o - P_a^2 = 0$$

If $h = 2.0 \times 10^{-3}$,

$$P_o^2 - 187.5 \; M_o P_o - P_a^2 = 0$$

If $h = 1.5 \times 10^{-3}$

$$P_o^2 - 333 \; M_o P_o - P_a^2 = 0$$

If $h = 1.0 \times 10^{-3}$

$$P_o^2 - 750 \; M_o P_o - P_a^2 = 0$$

If $h = 1.0 \times 10^{-3}$, let $M_o = 0.15$. Then

$$P_o = 112.5 \; P_o - P_a^2 = 0$$

$$P_o = \frac{112.5 + \sqrt{13,600 + 864}}{2} = \frac{112.5 + \sqrt{14,464}}{2} \; \frac{112.5 + 120.5}{2}$$

$$= 116.5$$

$$V_o = 1120 \times 0.15 = 167 \text{ ft/sec}$$

$$M_c = M_o \cdot \frac{2\pi R_o h}{\pi a^2} = \frac{.15 \times 4 \times 10^{-3} \times 2}{2.5 \times 10^{-3}} = 0.48$$

$$P_c/P_o = 1.17 \quad P_c = 136 \text{ too high}$$

Try

$$M_o = 0.14$$

$$P_o^2 - 105\ P_o - P_a^2 = 0$$

$$P_o = \frac{105 + \sqrt{11{,}000 + 864}}{2} = \frac{105 + \sqrt{11{,}864}}{2} = 107$$

$$M_c = M_o \frac{2\pi R_o h}{\pi a^2} = \frac{2 \times 4 \times 10^{-3}}{2.5 \times 10^{-3}} \quad M_o = 0.45$$

$$\frac{P_c}{P_o} = 1.15 \quad P_c = 123.5$$

If $h = 1.5 \times 10^{-3}$, let $M_o = 0.2$.

Therefore $$P_o^2 - 66.6\ P_o - P_a^2 = 0$$

$$P_o = \frac{66.6 + \sqrt{4440 + 864}}{2} = \frac{66.6 + \sqrt{5304}}{2} = \frac{66.6 + 73}{2} = 69.8$$

$$M_c = M_o \cdot \frac{2\pi R_o h}{\pi a^2} = \frac{0.2 \times 2 \times 4 \times 1.5}{2.5} = 0.96$$

$$\frac{P_a}{P_o} = 1.81$$

$$P_c = 1.81 \times 69.8 = 126 \text{ psi (close)}$$

It then seems that in this design the flows are choked for film thicknesses greater than 1.5×10^{-3} so that operating clearances will never be greater than this value.

Let us try $h = 1.25 \times 10^{-3}$. If $M_o = 0.17$,

$$P_o^2 - 82\ P_o - P_a^2$$

$$P_o = \frac{82 + \sqrt{6710 + 864}}{2} = \frac{82 + \sqrt{7574}}{2} = 84.5$$

$$M_c = \frac{0.17 \times 2 \times 4 \times 1.25}{2.5} = 0.68$$

$$\frac{P_c}{P_o} = 1.37,\ P_c = 1.37 \times 84.5 = 116$$

If $M_o = 0.18$,

$$P_o = \frac{87 + \sqrt{8364}}{2} = 89$$

$$M_c = 0.72$$

$$\frac{P_c}{P_o} = 1.41,\ P_c = 125$$

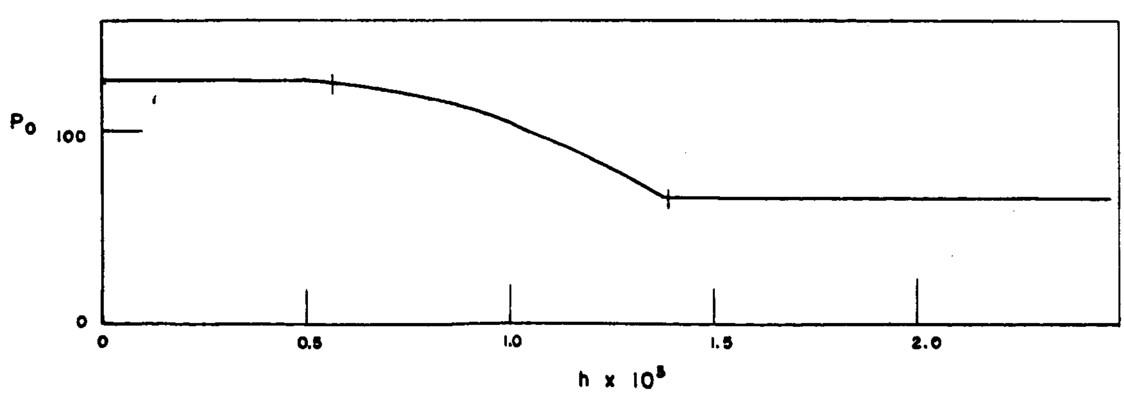


FIGURE 22

The load carrying capacity of the hydrostatic pad equals

$$P_o \pi R_o^2 + \int_{R_o}^{R_a} RP \cdot 2\pi dr$$

Since the dimension $(R_a - R_o)$ is smaller than R_o we will not be too far off if we assume a linear distribution of pressure across the area of the step bearing. Then,

$$W_p = P_o \pi R_o^2 + \pi(R_a^2 - R_o^2)\ P_o/2$$

$$W_p = \pi R_o^2 P_o + \pi \frac{R_a^2 P_o}{2} - \frac{\pi R_o^2 P_o}{2}$$

$$W_p = \frac{\pi P_o (R_o^2 + R_a^2)}{2}$$

$$R_o = 4 \qquad R_o^2 = 16$$

$$R_a = 5 \qquad R_o^2 = 25$$

$$\left(\frac{R_o^2 + R_a^2}{2}\right) \pi = \frac{41}{2} \times 3.14 = 64.5\ P_o$$

We can tabulate the load carrying capacity of the hydrostatic bearing vs. film thickness as follows:

$h \times 10^3$	P_o	W_s
1.0	107	6900 lb
1.25	84.5	5400 lb
1.5	69.8	4500 lb
2.0	66.0	4250 lb

If we refer to Figure 19 and 20, we see that the total load carrying capacity of each pad which is made up of two hydrostatic step bearings is

$$W_p = 2W_s \cos 22.5°$$

$h \times 10^3$	W_s	W_p
1.0	6900	12,740 lb
1.25	5400	9,960 lb
1.50	4500	8,300 lb
2.00	4250	7,850 lb
↓	4250	↓
	↓	

Figure 19 shows, however, that there are four such pads which make up the complete journal bearing. The net load carrying capacity of this bearing equals the resultant load carrying capacity of each pad.

If we move in the vertical direction with a machined radial clearance of 3.5×10^{-3}, then the sum of the film thicknesses of the top and bottom pads is 7.0×10^{-3}. If then the top pad is operating with a film thickness of 6.0×10^{-3}, the bottom pad film thickness must be 1.0×10^{-3}.

Then, if we move in a direction in line with the centers of a pair of opposed pads, we find

$h_1 \times 10^3$	$h_2 \times 10^3$	W_{p_1}	W_{p_2}	W_T
3.5	3.5	7850	7850	0
3.0	4.0	7850	7850	0
2.5	4.5	7850	7850	0
2.0	5.0	7850	7850	0
1.5	5.5	8300	7850	450 lb
1.25	5.75	9960	7850	2110 lb
1.0	6.0	12,740	7850	4890 lb

We should mention here that the bearing will probably have some additional load carrying capacity due to hydrodynamic action when operating films are small. Our experience tells us that the most we can hope to get hydrodynamically is about 5 psi over the projected area of the bearing. Since the area of each pad is 200 in.^{2}, the hydrodynamic action of the bearing would contribute about 1000 lb to the load carrying capacity of the bearing.

We can see from this table that the bearing has zero stiffness over a large part of the clearance. If we machined the bearing so that its radial clearance is 1.5×10^{-3}, we could eliminate the zero stiffness problem. However, now we have the problem of machining and maintaining a bearing 14 in. in diameter with a diametral clearance of 3.0×10^{-3} in., or compensating the bearings somehow. If the temperature of the shaft rose only 50°F the shaft would expand this much. Maximum flow of course would occur when the journal is centrally located and would equal

$$Q = 8V\pi a^2 \cdot \frac{P_o}{P_a} = 8(1120)(3.14) \times 2.5 \times 10^{-3} \times \frac{66}{14.7} 12 = 3790 \text{ in.}^3/\text{sec}$$

$$Q = 3790 \times 60 = 2.2 \times 10^5 \text{ in.}^3/\text{min or } 127.5 \text{ ft}^3/\text{min.}$$

Let us see what we can do to get us out of our problems of too small a clearance, too small a load carrying capacity, and too much air.

(1) Cut down the size of the orifice:

If we cut down the size of the orifice we can cut down on the quantity of flow but we decrease the percentage of the bearing clearance in which the bearing has some stiffness.

(2) Decrease the size of R_o:

This gives the bearing some stiffness over a greater part of its bearing clearance (or allows us to machine a larger clearance in the bearing) but it cuts down on the load carrying capacity of the bearing.

(3) Increase P_c to increase load carrying capacity:

If we increase P_c the quantity of standard air would increase. We would then have to decrease orifice size to cut back on the flow. If we cut back on the diameter of the orifice we increase the danger of clogging the orifice with foreign matter.

VI. SUMMARY

It seemed to us that the most important question to be answered in this program was whether air lubricated bearings could carry the loads encountered. There didn't seem to be much point in investigating the various types of instability which can be encountered with air bearings if it was found that they could not feasibly carry the loads encountered. Consequently, a survey was made to determine the actual loads imposed on the bearings and to find adequate design equations for them. Frankly, we did not have much success with either venture.

The bearing loads given to us by the engine manufacturers were considerably lower than those later calculated according to MIL-E-5007A.

Because of the fact that the highest of the maneuver loads are applied to the bearings for an extremely short time, they are not considered to affect the life of the rolling element bearings now used. It is because of this fact and because of the fact that air bearings cannot suffer an overload even for a brief instant that an analysis of typical bearing loads under the maneuvers listed in MIL-E-5007A was made.

Design formulae and methods for a simple step bearing were derived. The formulae include the effects of the acceleration of the gas as it passes through the bearing. These equations check the meager experimental data available more closely than any others yet derived.

Application of these equations to the particular case of aircraft turbojet engines shows that the use of gas bearings in this instance is not impossible. The analysis indicates very large bearings, small clearances, and large volumes of air will be required. It is our contention that further analysis backed up by experimentation should be undertaken to determine if the above requirements can be met.

We have spoken with several people of the Stratos Division of Fairchild Aircraft Corporation. We have been told that the high values of pressure required can be obtained with the use of a two-stage auxiliary

compressor even at the lowest values of inlet pressures available. The size of the unit required to give the desired flows, however, is not known. Stratos stated that such a compressor would involve its own development program.

We can then say that it is entirely possible to carry engine bearing loads with air lubricated bearings. The example discussed in this report, where 125 psia is required, shows that loads as high as about 5000 lbs. can be carried on a 14-in. diameter bearing. If the bearings could be made larger, if the maneuver loads on the bearings could be obtained more realistically than they are now, and if the required air pressure could be obtained, then air bearings could probably carry the loads required. Again, we would like to emphasize the many problems which would have to be solved before these bearings could be designed completely.

We have shown that it is theoretically possible to carry the heavy loads occurring. In order for us to do this, however, large recesses of high pressure are required. If these recesses are made shallow enough, they will, in general, cause no trouble. There is, however, one aspect of this design which is not yet fully understood. When the film thickness is large, the flow governing area is the area of the inlet orifice. As the load on the bearing increases, the film thickness decreases and the area determined by the outer periphery of the recess and the film thickness decreases proportionately. When the latter area approaches in size the fixed inlet orifice area, a type of instability is obtained. This type of instability is not completely understood. A further understanding of this phenomenon is required before heavily loaded air bearings can be designed with confidence. The bearings will have to be mounted in a self-compensating structure so that the small clearances required can be maintained throughout the operating temperature range of the bearing.

At just about the time that this feasibility study was to terminate, we received support from the office of Naval Research to create a general technology for gas bearings. The ultimate aim of this project is to place air bearings on a sound design basis. It is our hope that, in a short time, most of the problems attendant to air bearings will have been solved.

At any rate, we believe that there is definite hope for the use of air bearings for jet aircraft engines. The work done during this feasibility study will give us an invaluable start in the work for ONR.

George M. Robinson

George M. Robinson
Chief, Friction and
Lubrication Section

Approved by:

Frank S. Chaplin

Frank S. Chaplin
Assoicate Director
Mechanical Engineering Division

Nicol H. Smith

Nicol H. Smith
Director
Laboratories for
Research and Development.

APPENDIX

CALCULATION OF BEARING REACTIONS, R_1, R_2, and R_3

APPENDIX F-A1914

Calculation of Bearing Reactions, R_1, R_2 and R_3

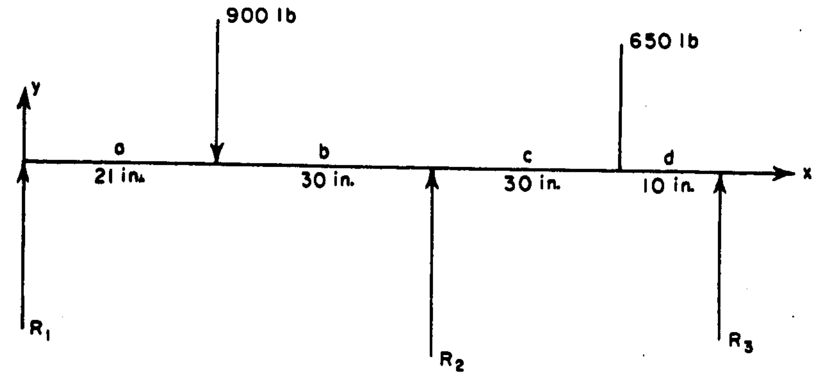


FIGURE 23

Section a

$$M = R_1 x$$

$$EI \frac{d^2y}{dx^2} = R_1 x$$

$$EI \frac{dy}{dx} = \frac{R_1 x^2}{2} + C_1$$

at x = 21

$$EI \frac{dy}{dx} = \frac{R_1(441)}{2} + C_1$$

Section b

$$M = R_1 x - 900\ (x - 21)$$

$$M = (R_1 - 900)\ x + 18{,}900$$

$$EI \frac{d^2y}{dx^2} = (R_1 - 900)\ x + 18{,}900$$

$$EI \frac{dy}{dx} = (R_1 - 900)\ \frac{x^2}{2} + 18{,}900\ x + C_2$$

at x = 21

$$EI \frac{dy}{dx} = (R_1 - 900)\ \frac{441}{2} + 18{,}900(21) + C_2$$

$$\frac{R_1(441)}{2} + C_1 = \frac{R_1(441)}{2} - 450\ (441) + 21(18{,}900) + C_2$$

$$EI\ y = \frac{R_1 x^3}{6} + C_1 x + C_3$$

at $x = 0$, $y = 0$, $C_3 = 0$

$$EI\ y = \frac{R_1 x^3}{6} + C_1 x$$

$$EI\ y = (R_1 - 900)\frac{x^3}{6} + \frac{18,900(x^2)}{2} + C_2 x + C_4$$

at $x = 51$, $y = 0$

$$EI\ y = (R_1 - 900)\frac{(x^3 - 133,000)}{6} + \frac{18,900}{2}(x^2 - 2601) + C_2 (x - 51)$$

at $x = 21$

$$EI\ y = \frac{R_1}{6}(9260) + 21\ C_1$$

at $x = 21$

$$EI\ y = (R_1 - 900)\frac{(9260 - 133,000)}{6} + \frac{18,900}{2}(441 - 2601) + C_2 (-30)$$

$$\frac{R_1}{6}(9260) + 21\ C_1 = (R_1 - 900)\frac{(-123,740)}{6} + \frac{18,900\ (-2160)}{2} - 30\ C_2$$

$$1543\ R_1 + 21\ C_1 = (R_1 - 900)(-20,623) - 18,900\ (1080) - 30(C_1 - 199,000)$$

$$1543\ R_1 + 21\ C_1 = -20,623\ R_1 + 18.56 \times 10^6 - 20.41 \times 10^6 - 30\ C_1 + 5.97 \times 10^6$$

$$51\ C_1 = 4.14 \times 10^6 - 22,166\ R_1$$

$$C_2 = C_1 - 199,000$$

$$C_2 = \frac{4.14 \times 10^6 - 22,166\ R_1 - 10.15 \times 10^6}{51}$$

Section C

$$EI \frac{d^2y}{dx^2} = R_1 (x) + R_2(x - 51) -900 (x-21) = R_1(x) + R_2(x)-51 R_2 - 900x + 18,900$$

$$EI \frac{dy}{dx} = (R_1 + R_2 - 900) \frac{x^2}{2} - (51 R_2 - 18,900)x + C_5$$

$$EI\ y = (R_1 + R_2 - 900) \frac{x^3}{3} - (51 R_2 - 18,900) \frac{x^2}{2} + C_5x + C_6$$

at $x = 51$

$$EI \frac{dy}{dx} = (R_1 + R_2 - 900) \frac{2601}{2} - (51 R_2 - 18,900)\ 51 + C_5$$

but from Section b at $x = 51$

$$EI \frac{dy}{dx} = (R_1 - 900) \frac{2601}{2} + 18,900\ (51) - \frac{6.01 \times 10^6 - 22,166 R_1}{51}$$

therefore

$$1300 R_2 - 2601 R_2 + C_5 = 118,000 - 434 R_1$$

$$434 R_1 - 1301 R_2 + C_5 = 118,000$$

at $x = 51$, $y = 0$

$$(R_1 + R_2 - 900) \frac{(51)^3}{6} - (51 R_2 - 18,900) \frac{(51)^2}{2} + C_5 (51) + C_6 = 0$$

$$C_6 = -(R_1 + R_2 - 900) \frac{(133,000)}{6} + (51 R_2 - 18,900) \frac{2601}{2} - 51 C_5$$

therefore

$$EI\ y \text{ (at section C)} = \frac{(R_1+R_2-900)}{6}(x^3 - 133,000) - \frac{(51 R_2 - 18,900}{2} (x^2-2601) +$$

$$C_5 (x-51)$$

at $x = 81$

$$EI\ y = 53{,}400\ R_1 - 4{,}300\ R_2 - 18.5 \times 10^6$$

$$EI\ \frac{dy}{dx} = 2846\ R_1 + 451\ R_2 - 1.30 \times 10^6$$

Section d

$$EI\ \frac{d^2y}{dx^2} = R_1 x + R_2\ (x-51) - 900\ (x-21) - 650\ (x-81)$$

$$= R_1 x + R_2 x - 51\ R_2 - 900x + 18{,}900 - 650x + 52{,}700$$

$$= (R_1 + R_2 - 1550)x - (51\ R_2 - 71{,}600)$$

$$EI\ \frac{dy}{dx} = (R_1 + R_2 - 1550)\ \frac{x^2}{2} - (51\ R_2 - 71{,}600)\ x + C_7$$

$$EI\ y = (R_1 + R_2 - 1550)\ \frac{x^3}{6} - (51\ R_2 - 71{,}600)\ \frac{x^2}{2} + C_7 x + C_8$$

at $x = 81$

$$EI\ \frac{dy}{dx} = (R_1 + R_2 - 1550)\ \frac{6561}{2} - (51\ R_2 - 71{,}600)\ (81) + C_7$$

$$= 3280\ R_1 + 3280\ R_2 - 5.08 \times 10^6 - 4130\ R_2 + 5.8 \times 10^6 + C_7$$

$$= 3280\ R_1 - 850\ R_2 + 720{,}000 + C_7$$

From Section C

$$EI \frac{dy}{dx}_{at\ x=81} = 2846\ R_1 + 451\ R_2 - 1.30 \times 10^6$$

therefore

$$3280\ R_1 - 850\ R_2 + 720{,}000 + C_7 = 2846\ R_1 + 451\ R_2 - 1.30 \times 10^6$$

at x = 81

Section d

$$EI\ y = (R_1+R_2-1550)\ \frac{(531{,}00)}{6} - (51\ R_2-71{,}600)\ \frac{(6561)}{2} + (1301\ R_2-434\ R_1-208\text{x}10^6)81+$$

$$C_8 = 53{,}400\ R_1 + 27{,}000\ R_2 - 70.5 \times 10^6 + C_8$$

Section C	Section d
at x = 81	at x = 81

$$EI_y = 53{,}400\ R_1-4300\ R_2-18.5\text{x}10^6 = 53{,}400\ R_1+27{,}000\ R_2-70.5\text{x}10^6 + C_8$$

Section d

$$EI\ y = (R_1+R_2-1550)\ \frac{x^3}{6} - (51\ R_2-71{,}600)\ \frac{x^2}{2} + (1301-434\ R_1-2.08\text{x}10^6)x +$$

$$52 \times 10^6 - 31{,}300\ R_2$$

at x = 91, y = 0

$$0 = (R_1+R_2-1550)\ (125,500) - (51\ R_2-71,600)\ (4140) +$$

$$(1301-434\ R_1 - 2.08 \times 10^6)\ 91 + 52 \times 10^6 - 31,300\ R_2$$

$$\Sigma M_{R_3} = 0 = R_1\ (91) + R_2(40) - 900(70) - 650(10) = 91\ R_1 + 40\ R_2 - 69,500$$

$$860\ R_1 + 7\ R_2 - 360 \times 10^3 = 0$$

$$91\ R_1 + 40\ R_2 - 69 \times 10^3 = 0$$

From the above two simultaneous equations we find

$$R_1 = 414 \text{ lb}$$

$$R_2 = 795 \text{ lb}$$

$$R_3 = 1550 - 795 - 414 = 341 \text{ lb}$$

therefore

$$R_1 = 414 \text{ lb}$$

$$R_2 = 795 \text{ lb}$$

$$R_3 = 341 \text{ lb}$$

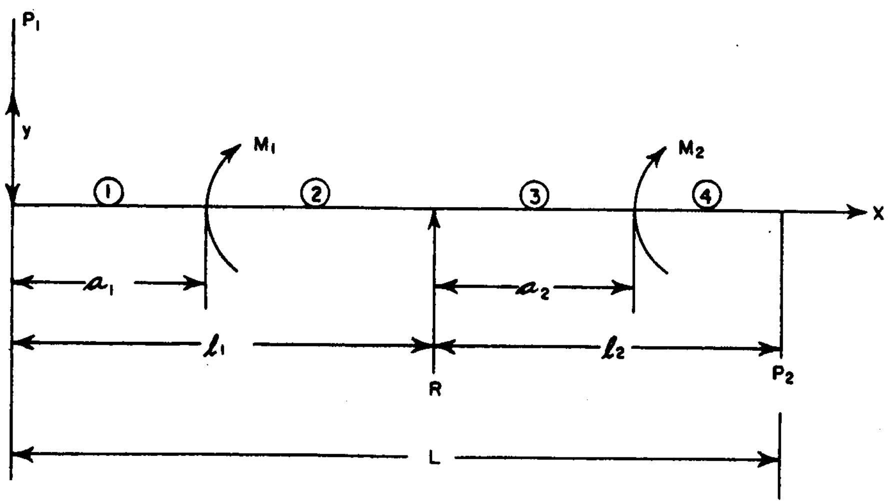


FIGURE 24

The three Equations which will be used are

$$\Sigma F\ y = 0 = -P_1 + R + P_2$$

$$\Sigma M_2 = 0 = -P_1\ L + M_1 + M_2 + R\ell_2 \text{ or } P_1 = \frac{M_1 + M_2 + R\ell_2}{L}$$

$$\frac{\partial \mu}{\partial R} = 0 = \int_0^L M_x \frac{\partial M x}{\partial R}\ dx$$

therefore

$$\frac{\partial P_1}{\partial R} = \frac{\ell_2}{L}$$

Section 1

$$M_x = -P_1 x$$

$$\frac{\partial M_x}{\partial R} = -x \frac{\partial P_1}{\partial R} = -x \frac{\ell_2}{L}$$

Section 2

$$M_x = -P_1 x + M_1$$

$$\frac{\partial M_x}{\partial R} = -x \frac{\ell_2}{L}$$

Section 3

$$M_x = -P_1 x + M_1 \; R(x - \ell_1)$$

$$\frac{\partial M_x}{\partial R} = -x \frac{\ell_2}{L} + (x-\ell_1)$$

Section 4

$$M_x = -P_1 x + M_1 + R\,(x-\ell_1) + M_2$$

$$\frac{\partial M_x}{\partial R} = -x \frac{\ell_2}{L} + (x-\ell_1)^1$$

Section 1

$$\frac{\partial \mu_1}{\partial R} = \int_o^{a_1} -P_1(x)\,(-x)\,\frac{\ell_2}{L}\,dx = \int_o^{a_1} P_1 x^2 \frac{\ell_2}{L}\,dx = P_1 \frac{\ell_2}{L}\frac{x^3}{3}\Bigg|_o^{21} =$$

$$\frac{P_1}{3}\,\frac{40}{91}\,(9261) = 1352\;P_1$$

$$\frac{\partial\mu_2}{\partial R} = \int_{a_1}^{\ell_1} (-P_1x+M_1)\ (-x)\ \frac{\ell_2}{L} = \int_{a_1}^{\ell_1} P_1x^2\ \frac{\ell_2}{L}\ dx - \int_{a_1}^{\ell_1} M_1x\ \frac{\ell_2}{L}\ dx$$

$$\frac{\partial\mu_2}{\partial R} = P_1\ \frac{\ell_2}{L}\ \frac{x^3}{3}\Bigg|_{21}^{51} - M_1\ \frac{\ell_2}{L}\ \frac{x^2}{2}\Bigg|_{21}^{51} = 18{,}100\ P_1\ - 475\ M_1$$

$$\frac{\partial\mu_3}{\partial R} = \int_{\ell_1}^{\ell_1+a_2} \left[-P_1x+M_1\ + \frac{(P_1L-M_1-M_2)}{\ell_2}\ \ (x-\ell_1)\right]\left[-x\ \frac{\ell_2}{L} + (x-\ell_1)\right] dx$$

$$= \int_{\ell_1}^{\ell_1+a_2} \left[-P_1x+M_1+\frac{P_1L}{\ell_2}\ x - \frac{M_1x}{\ell_2} - \frac{M_2x}{\ell_2}\ - \frac{P_1L\ell_1}{\ell_2}\ + \frac{M_1\ell_1}{\ell_2} + \frac{M_2\ell_1}{\ell_2}\right]\left[-x\ \frac{\ell_2}{L}\ +(x-\ell_1)\right] dx$$

$$\frac{\partial\mu_3}{\partial R} = \frac{P_1x^3\ell_2}{3L} - \frac{M_1x^2\ell_2}{2L} - \frac{P_1x^3}{3} + \frac{M_1x^3}{3L} + \frac{M_2x^3}{3L} + \frac{P_1\ell_1x^2}{2} - \frac{M_1\ell_1x^2}{2L}$$

$$- \frac{M_2\ell_1x^2}{2L} - \frac{P_1x^3}{3} + \frac{P_1x^2\ell_1}{2} + \frac{M_1x^2}{2} - M_1\ell_1x + \frac{P_1Lx^3}{3\ell_2} - \frac{P_1L\ell_1x^2}{2\ell_2} -$$

$$\frac{M_1x^3}{3\ell_2} + \frac{M_1x^2\ell_1}{2\ell_2} - \frac{M_2x^3}{3\ell_2} - \frac{P_1L\ell_1x^2}{2\ell_2} + \frac{P_1L\ell_1^2x}{\ell_2} + \frac{M_2x^2\ell_1}{2\ell_2}\ +$$

$$\left.\frac{M_1\ell_1x^2}{2\ell_2}\ - \frac{M_1\ell_1x^2}{\ell_2} + M_2\ \frac{\ell_1}{\ell_2}\ \frac{x^2}{2} - M_2\ \frac{\ell_1^2}{\ell_2}\ \ x\ \right]_{51}^{81}$$

$$\frac{\partial\mu_3}{\partial R} = P_1\ [58{,}500-266{,}000+204{,}000+303{,}000-458{,}000+177{,}500\ ]\ +$$

$$M_1\ [-878+1461-1119+1980-1530-3330+5090-1950\ ]+$$

$$M_2\ \ [1462-1119-3330+5090-1950\ ]$$

$$\frac{\partial\mu_4}{\partial R} = \int_{\ell_1+a_2}^{L} \left[-P_1x+M_1+M_2+R\ (x-\ell_1)\right] \left[-x\,\frac{\ell_2}{L} + (x-\ell_1)\right] dx$$

$$\frac{\partial\mu_4}{\partial R} = \Bigg[\frac{P_1x^3\ell_2}{3L} - \frac{M_1x^2\ell_2}{2L} - \frac{P_1x^3}{3} + \frac{M_1x^3}{3L} + \frac{M_2x^3}{3L} + \frac{P_1\ell_1x^2}{2} - \frac{M_1\ell_1x^2}{2L} - \frac{M_2\ell_1x^2}{2L}$$

$$- \frac{P_1x^3}{3} + \frac{P_1x^2\ell_1}{2} + \frac{M_1x^2}{2} - M_1\ell_1x + \frac{P_1Lx^3}{3\ell_2} - \frac{P_1L\ell_1x^2}{2\ell_2} - \frac{M_1x^3}{3\ell_2} + \frac{M_1x^2\ 1}{2\ell_2}$$

$$- \frac{M_2x^3}{3\ell_2} - \frac{P_1L\ell_1x^2}{2\ell_2} + \frac{P_1L\ell_1^2x}{\ell_2} + \frac{M_2x^2\ell_1}{2\ell_2} + \frac{M_1\ell_1x^2}{2\ell_2} - \frac{M_1\ell_1^2x}{\ell_2} +$$

$$+ M_2\,\frac{\ell_1}{\ell_2}\,\frac{x^2}{2} - M^2\,\frac{\ell_1^2x}{\ell_2} - M_2\,\frac{\ell_2}{L}\,\frac{x^2}{2} + \frac{M_2x^2}{2} - M_2\ell_1x\ \Bigg]_{\ell_1+a_2=81}^{L=91}$$

$$\frac{\partial\mu_4}{\partial R} = \frac{P_1\ (222{,}130)\ (40)}{3(91)} - \frac{M_1\ (40)\ (1720)}{2(91)} - \frac{2\ P_1\ (222{,}130)}{3} + \frac{M_1\ (222{,}130)}{273}$$

$$+ \frac{M_2(222{,}130)}{273} + P_1\ (1720)\ (51) - \frac{M_1\ (51)\ (1720)}{182} - \frac{M_2\ (51)\ (1720)}{182}$$

$$+ \frac{M_1\ (1720)}{2} - M_1\ (51)\ (10) + \frac{P_1\ (91)\ (222{,}130)}{3\ (40)} - \frac{P_1\ (91)\ (51)\ (1720)}{2\ (40)}$$

$$- \frac{M_1\ (222{,}130)}{3\ (40)} + \frac{M_1\ (1720)\ (51)}{80} - \frac{M_2\ (222{,}130}{120} - \frac{P_1\ (91)\ (51)\ (1720)}{80}$$

$$+ \frac{P_1\ (91)\ (10)\ (2601)}{40} + \frac{M_2\ (1720)\ (51)}{80} + \frac{M_1\ (51)\ (1720)}{80} - \frac{M_1\ (2601)\ (10)}{40}$$

$$+ M_2\,\frac{51}{80}\,(1720) - M_2\,\frac{2601}{40}\,(10) - M_2\,\frac{40}{91}\,(\frac{1720}{2}) + M_2\,(\frac{1720}{2}) - M_2\ (51)\ (10)$$

$$\frac{\partial \mu_4}{\partial R} = 800\ P_1 - 4M_1 - 4M_2$$

Therefore

$$\frac{\partial \mu}{\partial R} = P_1 [1352+18,100+20,500+800] + M_1 [-475-276-4] + M_2\ [132-4] = 0$$

$$40,752\ P_1 + 128\ M_2 - 755\ M_1 = 0$$

During the yaw maneuver at 4.0 rad/sec, with the engine running at 7500 rpm, M_1 = 750,000 in.lb and M_2 = 610,000 in.lb.

$$P_1 = \frac{755\ M_1 - 128\ M_2}{40,752} = \frac{755(750,000)-128(610,000)}{40,752}$$

$$P_1 = \frac{(567-78) \times 10^6}{40,752} = \frac{489 \times 10^6}{40.75 \times 10^3} = 12,000 \text{ lb}$$

$$R = \frac{P_1 L - M_1 - M_2}{\ell_2} = \frac{12,000(91) - 1,360,000}{40} = \frac{270,000}{40} = -\ 6,750 \text{ lb}$$

$$P_2 = P_1 - R = 12,000-(-6750) = 18,750 \text{ lb}$$

DISTRIBUTION LIST

No. of Copies	
20	National Advisory Committee on Aeronautics 1512 H Street, N.W. Washington 25, D. C. Attn: Mr. Ira H. Abbott Assistant Director for Research

THE
FRANKLIN INSTITUTE
LABORATORIES FOR RESEARCH AND DEVELOPMENT
PHILADELPHIA 3, PA.